A thin uniform tube is bent into a circle of radius r in the vertical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and ρ_2 ($\rho_1 > \rho_2$) fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is:

A $\quad \theta = \tan^{-1}\left[\dfrac{\pi}{2}\left(\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)\right]$

B $\quad \theta = \tan^{-1}\dfrac{\pi}{2}\left(\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$

C $\quad \theta = \tan^{-1}\pi\left(\dfrac{\rho_1}{\rho_2}\right)$

D $\quad$ None of above

Solution

Let us find the pressure at the lowest point 1. Since the liquid has density

ρ_2 and height

h_2' on the right hand side of point 1, we have

$P_1 = \rho_1 g h_1$ ....(1)

Since two liquid columns of height $h_1\ and\ h_2$ and densities $\rho_1\ and\ \rho_2$ are situated above point 1, on the left-hand side, we have

$P_2 = \rho_1 g h_2 + \rho_2 g h_2'$ ....(2)

Equating $P_1 and P_2$, we get

$\rho_1 h_2 + \rho_2 h_2' = \rho_1 h_1$
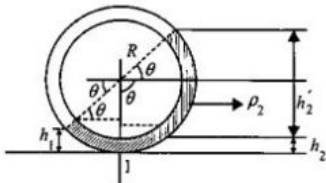
Substituting

$h_2' = R\sin\theta + R\cos\theta,\ h_2 = R(1 - \cos\theta)\ and\ h_1 = R(1 - \sin\theta)$

$\rho_1 R(1 - \cos\theta) + \rho_2 R(\sin\theta + \cos\theta) = \rho_1 R(1 - \sin\theta)$

This gives

$\tan\theta = \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is :

A $\quad \dfrac{2}{3}\alpha$

B $\quad \dfrac{5}{2}\alpha$

C $\quad \dfrac{3}{2}\alpha$

D $\quad \alpha$

Solution

S=

$4\pi R^2$

$\ln S = \ln(4\pi) + \ln(R^2)$

$\ln S = 2\ln R$

$\dfrac{\Delta S}{S} = 2\dfrac{\Delta R}{R} = \alpha$

$\dfrac{\Delta R}{R} = \dfrac{\alpha}{2}$ ------------(1)

V= $\dfrac{4}{3}\pi R^3$
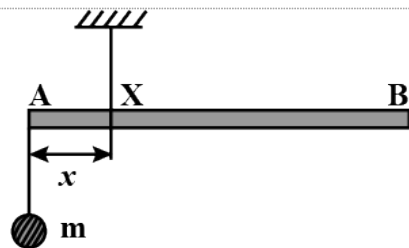
$\ln V = \ln(\dfrac{4}{3}\pi) + \ln R^3$

$\ln V = 3 \ln R$

$\dfrac{\Delta V}{V} = 3\dfrac{\Delta R}{R}$

$\dfrac{\Delta V}{V} = 3(\dfrac{\alpha}{2})$

#868156

A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variables that give a straight line, when plotted, are:

A $m, \dfrac{1}{x}$

B $m, \dfrac{1}{x^2}$

C m, x

D m, x^2

Solution

Let, 'l' and 'M' be the length and mass of rod.

for equilibrium, taking moment about X, we get

$m \times x = M \times (\dfrac{l}{2} - x)$

this can be written as,

$m = (\dfrac{Ml}{2})\dfrac{1}{x} - M$

comparing with straight line equation,

y = mx + c

we can say that,

$m \propto \dfrac{1}{x}$

#868160

An ideal capacitor of capacitance $0.2\mu F$ is charged to a potential difference of $10V$. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance $0.5mH$. The current at a time when the potential difference across the capacitor is $5V$, is:

A $0.17A$

B $0.15A$

C $0.34A$

D $0.25A$

Solution

LC circuit

$\Rightarrow V_C + V_L = 0$ and $I_C - I_L$

$V_2 = \dfrac{L_d I_L}{dt}$ ---------- (1)

$\Rightarrow q = CV_c$

$\Rightarrow \dfrac{d_v}{d_t} = \dfrac{dV_c}{dt}$

$\Rightarrow I_c = \dfrac{C_d V_c}{dt}$ ----------(2)

Now , $I_c = I_L = I$

So, Salving (1) and (2),

$\Rightarrow \dfrac{d^2 I}{dt^2} + w_o^2 I = O$

$\Rightarrow I = I_0 sin(w_o t)$ where $w_o = \dfrac{1}{\sqrt{LC}}$

$\Rightarrow V_L = L\dfrac{dI}{dt} = LI_o w_o w_s (w_o t)$

$At\, t = 0, V_L = 10V,$

Hence $LI_o w_o = 10V$

at some time , where $V_L = 5v$

t is such that $\cos(w_o t) = \dfrac{1}{2}$

$\Rightarrow cos^2(wot) + sin^2(wot) = 1$

$\Rightarrow sin(wot) = \sqrt{1 - Y_4} = \dfrac{\sqrt{3}}{2}$

Now , $LI_o w_o = 10$

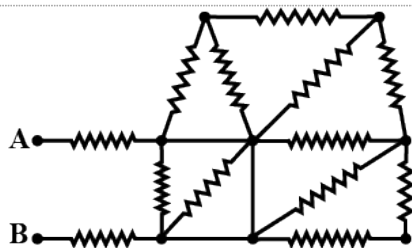$\Rightarrow I_o = \dfrac{10}{w_o L} = 10\sqrt{\dfrac{C}{L}} = 0.2$

So at that time

$I = I_o \dfrac{\sqrt{3}}{2}$

$0.2 \times \dfrac{\sqrt{3}}{2} = 0.17$

Ans is (1)

#868162

In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is:

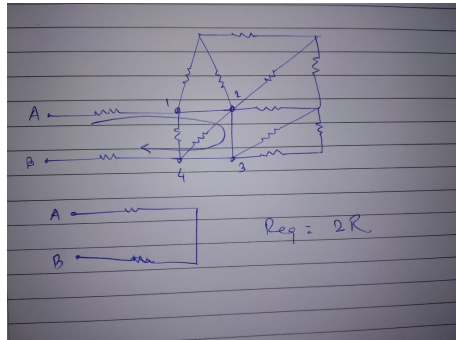| A	$2R$

B	$\dfrac{5R}{2}$

C $\dfrac{5R}{3}$

D $3R$

Solution

Point 1, 2, 3 and 4 are same potential (shot circuited),hence equivalent resistance becomes

$R_{Eq} = R + R = 2R$

#868168

In a common emitter configuration with suitable bias, it is given than R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by:

β is current gain, I_B, I_C, I_E are respectively base, collector and emitter currents:

A $\beta \dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_E}{\Delta I_B}, \beta^2 \dfrac{R_L}{R_{BE}}$

B $\beta^2 \dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_B}, \beta \dfrac{R_L}{R_{BE}}$

C $\beta^2 \dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_E}, \beta^2 \dfrac{R_L}{R_{BE}}$

D $\beta \dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_B}, \beta^2 \dfrac{R_L}{R_{BE}}$
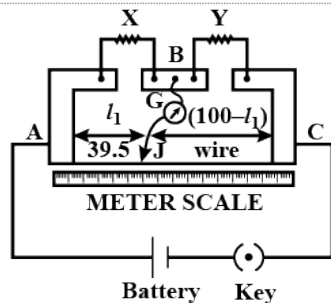
Solution

Voltage gain =

$\dfrac{V_{CE}}{V_{BE}} = \beta \dfrac{R_L}{R_{BE}}$

Current gain = $\beta = \dfrac{I_C}{I_B}$

Power gain = $voltage\ gain \times current\ gain = \beta^2 \dfrac{R_L}{R_{BE}}$

#868172

In a meter bridge, as shown in the figure, it is given that resistance $Y = 12.5\Omega$ and that the balance is obtained at a distance $39.5cm$ from end A (by jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance l_2 from end A. What are the values of X and l_2 ?

A 19.15Ω and $39.5cm$

B 8.16Ω and $60.5cm$

C 19.15Ω and $60.5cm$

D 8.16Ω and $39.5cm$

Solution

$$\frac{X}{Y} = \frac{39.5}{60.5}$$

$$X = \frac{39.5}{60.5} \times 12.5$$

$$= 8.16\Omega$$

$$\frac{12.5}{8.16} = \frac{l_2}{100 - l_2}$$

$$1.53(100 + l_2) = l_2$$

$$153 - 1.53\, l_2 = l_2$$

$$\frac{153}{2.53} = l_2$$

$$= 60.5cm$$

#868175

Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2, their de Broglie wavelength in the frame of reference attached to their centre of mass is:

A $\lambda_{CM} = \lambda_1 = \lambda_2$

B $\dfrac{1}{\lambda_{CM}} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

C $\lambda_{CM} = \dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1{}^2 + \lambda_2{}^2}}$

D $\lambda_{CM} = \left(\dfrac{\lambda_1 + \lambda_2}{2}\right)$

Solution

Since

$$\lambda = \frac{h}{mv}$$
$$v = \frac{h}{m\lambda}$$

Let v_1 and v_2 are the speeds of electrons

$$v_{cm} = \frac{v_1 + v_2}{2}$$
$$\frac{h}{2m\lambda_{cm}} = \frac{1}{2}\left(\frac{h}{m\lambda_1} + \frac{h}{m\lambda_2}\right)$$

$$\frac{1}{\lambda_{cm}} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

#868177

A tuning fork vibrates with frequency $256Hz$ and gives one best per second with the third normal mode of vibration of an open pipe. What is the length of the pipe? (Speed of sound of air is $340ms^{-1}$)

A $190cm$

B $180cm$

C $220cm$

D $200cm$

Frequency of tuning fork = 256 Hz

Frequency of open pipe = 256 ± 1 Hz

Frequency of third normal mode of vibration of an open pipe is $\nu = \dfrac{3v}{2l}$

here l = length of pipe

$\dfrac{3 \times 340}{2 \times l} = 255$

$l = 200cm$

#868180

The number of amplitude modulated broadcast stations that can be accommodated in a $300kHz$ band width for the highest modulating frequency $15kHz$ will be:

A   20

B   10

C   8

D   15

Solution

$Number\ of\ Stations = \dfrac{Band\ Width}{2 \times Highest\ Modulating\ Frequency}$

$Number\ of\ Stations = \dfrac{300kHz}{2 \times 15kHz}$

$Number\ of\ Stations = 10$

#868186

A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\dfrac{R}{2}$, and the other mass, in a circular orbit of radius $\dfrac{3R}{2}$. The difference between the final initial total energies is:

A $-\dfrac{GMm}{2R}$

B $+\dfrac{GMm}{6R}$

C $-\dfrac{GMm}{6R}$

D $\dfrac{GMm}{2R}$

Solution

Initial energy =

$-\dfrac{GMm}{R} + \dfrac{1}{2}\dfrac{GMm}{R} = -\dfrac{GMm}{2R}$

Final energy $= -\dfrac{GMm/2}{2R/2} - \dfrac{-GMm/2}{2 \times 3R/2}$

$\Delta E = -\dfrac{GMm}{6R}$

#868188

Light wavelength $550nm$ falls normally on a slit of width $22.0 \times 10^{-5}cm$. The angular position of the second minima from the central maximum will be (in radians)

π

A	$\dfrac{\pi}{8}$
B	$\dfrac{\pi}{12}$
C	$\dfrac{\pi}{4}$
D	$\dfrac{\pi}{6}$

Solution

for single slit, the angular position is given as
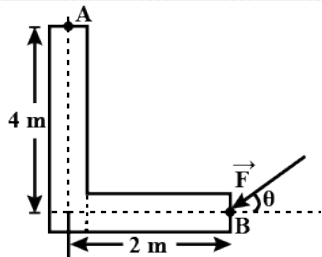
$$sin\theta = \frac{n\lambda}{d}$$

Here, n = 2

$$sin\theta = \frac{2 \times 550 \times 10^{-9}}{22 \times 10^{-7}}$$
$$sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}$$

#868191

A force of $40N$ acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by:

A	$\tan\theta = \dfrac{1}{4}$
B	$\tan\theta = 2$
C	$\tan\theta = \dfrac{1}{2}$
D	$\tan\theta = 4$

Solution

Moment of force about point A will be given as

$$\tau = Fcos\theta \times 4 + Fsin\theta \times 2$$

For maximu τ,

$$\frac{d\tau}{d\theta} = 0$$
$$0 = -4Fsin\theta + 2Fcos\theta$$
$$\tan\theta = \frac{1}{2}$$

#868193

A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction

between the object and the incline is :

A	$\sqrt{1 - \dfrac{1}{n^2}}$
B	$1 - \dfrac{1}{n^2}$
C	
	1

Solution

For a body moving with constant acceleration, the kinematics equation is

$$s = ut + \tfrac{1}{2}at^2$$

If the initial speed is zero, then the time taken to reach a distance s is $t = \sqrt{\dfrac{2s}{a}}$
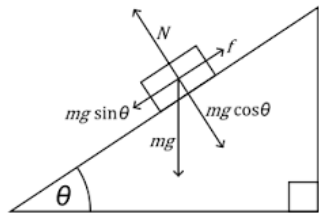
i.e., $t \propto a^{-0.5}$

In the case of a smooth inclined plane, $a_1 = g\sin\theta$

In the case of rough inclined plane, $a_2 = g(\sin\theta - \mu\cos\theta)$
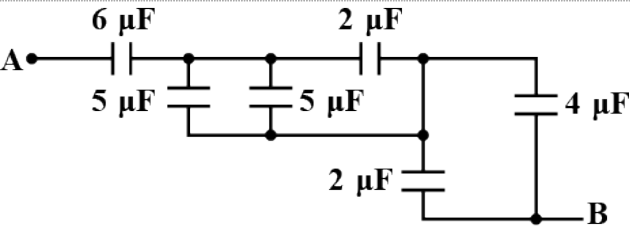
Time taken to travel down the smooth inclined plane is $t_1 = t$

Time taken to travel down the smooth inclined plane is $t_2 = nt$

$$\dfrac{t_1}{t_2} = \sqrt{\dfrac{a_2}{a_1}} \Rightarrow \dfrac{t_1^2}{t_2^2} = \dfrac{a_2}{a_1} \Rightarrow \dfrac{1}{n^2} = \dfrac{\sin\theta - \mu\cos\theta}{\sin\theta} \Rightarrow \mu = \tan\theta\left(1 - \dfrac{1}{n^2}\right)$$

#868195

The equivalent capacitance between A and B in the circuit given below is:

A $4.9\mu F$

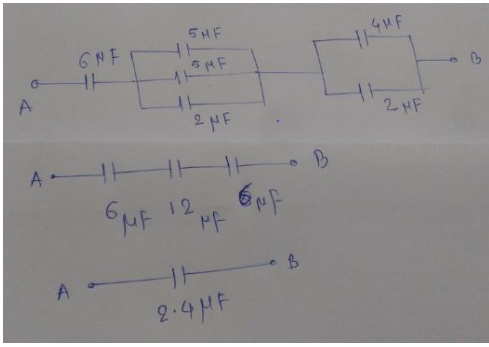**B** $3.6\mu F$

C $5.4\mu F$

D $2.4\mu F$

Solution

$C_1 = 5 + 5 + 2 = 12\mu F$

$C_2 = 4 + 2 = 6\mu F$

$1/C_{Eq} = 1/6 + 1/12 + 1/6 = 5/12$

$C_{Eq} = 2.4\mu F$

#868198

A Carnot's engine works as a refrigerator between $250K$ and $300K$. It receives $500cal$ heat from the reservoir at the lower temperature.The amount of work done in each cycle to operate the refrigerator is:

A $420J$

B $2100J$

C $772J$

D $2520J$

Solution

COP =
$$\frac{T_1}{T_2 - T_1} = \frac{250}{300 - 250} = 5$$
$$COP = 5 = \frac{Q_L}{W}$$
$$W = \frac{500 \times 4.184}{5} = 420J$$

#868200

One mode of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27^oC. The work done on the gas will be:

A $300R \ln 6$

B $300R$

C $300R \ln 7$

D $300R \ln 2$

Solution

Work done in isothermal process is given as
$$W = -nRT \; ln\frac{V2}{V_1}$$
$$W = R \times 300 \; ln\frac{P_1}{P_2}$$
$$W = -R \times 300 \; ln\frac{P}{2P}$$
$$W = 300R \; ln2$$

#868201

An automobile, travelling at $40km/h$, can be stopped at a distance of $40m$ by applying brakes. If the same automobile is travelling at $80km/h$, the minimum stopping distance, in metres, is (assume no skidding)

A $75m$

B $160m$

C $100m$

D $150m$

Solution

Since stopping distance,

$$s = \frac{v^2}{2a}$$

For case 1, $40 = \dfrac{(40 \times \frac{5}{18})^2}{2a}$ ....(1)

For case 2, $s = \dfrac{(80 \times \frac{5}{18})^2}{2a}$ ....(2)

From (1) and (2), $s = 160m$

#868207

Take the mean distance of the moon and the sun from the earth to be $0.4 \times 10^6 km$ and $150 \times 10^6 km$ respectively. Their masses are $8 \times 10^{22} kg$ and $2 \times 10^{30} kg$ respectively. The radius of the earth is $6400 km$. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\dfrac{\Delta F_1}{\Delta F_2}$ is:

A   2

B   6

C 10^{-2}

D   0.6

Solution

$\Delta F_1 = F_1 - F_2$

$\Delta F_2 = F_1' - F_2'$

$$\Delta F_1 = \frac{Gm_m m_e}{(0.4 \times 10^6 - Re)} - \frac{Gm_m m_e}{(0.4 \times 10^6 + Re)}$$

$$\Delta F_2 = \frac{Gm_m m_s}{(150 \times 10^6 - Re)} - \frac{Gme_m m_e}{(150 \times 10^6 + Re)}$$

$$\frac{\Delta F_1}{F_2} = \frac{GM_m Me \left\{ \dfrac{1}{0.4 \times 10^6 - Re} - \dfrac{1}{0.4 \times 10^6 + Re} \right\}}{GM_e Ms \left\{ \dfrac{1}{150 \times 10^6 - Re} - \dfrac{1}{150 \times 10^6 + Re} \right\}}$$

$$= \frac{\dfrac{Mm}{Ms} \left\{ \dfrac{0.4 \times 10^6 + Re - 0.4 \times 10^6 + Re}{(0.4 \times 10^6 - Re)(0.4 \times 10^6 + Re)} \right\}}{\left\{ \dfrac{150 \times 10^6 + Re - 150 \times 10^6 + Re}{(150 \times 10^6 - Re)(150 \times 10^6 + Re)} \right\}}$$

$$\frac{Mm}{Ms} \times \frac{(150 \times 10^6 - Re)(150 \times 10^6 + Re)}{(0.4 \times 10^6 - Re)(0.4 \times 10^6 + Re)} = \frac{(150 - .0064)(150 + .0064)}{(0.4 - .0064)(0.4 + .0064)}$$

$$= \frac{Mm}{Ms} \frac{149.99 \times 150.0064}{0.3936 \times .4064}$$

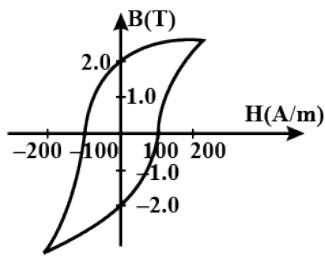$$= 140657.633 \times \frac{8 \times 10^{22}}{2 \times 10^{30}}$$

≈ 0.0056

≈ 0.01

$\approx 10^{-2}$

#868212

The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a ling solenoid with 1000 turns/cm.. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is:

A $2mA$

B $1mA$

C $40\mu A$

D $20\mu A$
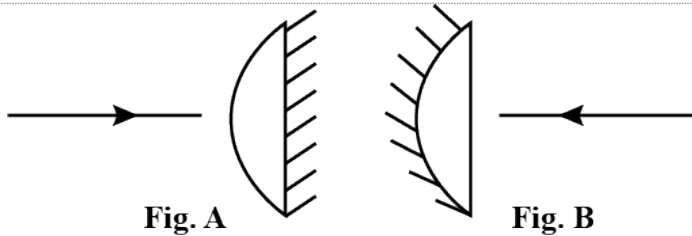
Solution

$N = 1000 turns/an$

$$H = \frac{NI}{l}$$

$$100\frac{A}{M} = \frac{1000 \times I}{10^{-2}}$$

$$100A = 10^5 \times I$$

$$10^{-3}A = I$$

$$I = 1mA$$

#868214

Fig. A   **Fig. B**

A planoconvex lens becomes an optical system of $28cm$ focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A.

If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig. B, it acts like an optical system of focal length $10cm$. The refractive index of the material of lens if:

A 1.50

B 1.55

C 1.75

D 1.51

Solution

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$
$f_m \implies \infty$
This gives, $f = 56$
$$f_m = \frac{R}{2}\backslash$$
Hence
$$\frac{1}{F} = \frac{2}{f} + \frac{2}{R}$$

This gives, $R = 31.11$

$$\frac{1}{f} = (\mu - 1)(\frac{1}{R})$$

$\mu = 1.55$

#868216

A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at $x = 0$, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct?

A   The total energy of the system is

$$\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$$

B   The new equilibrium position is at a distance: $\dfrac{2qE}{k}$ from $x = 0$

C   The new equilibrium position is at a distance: $\dfrac{qE}{2k}$ from $x = 0$

D   The total energy of the system is

$$\frac{1}{2}m\omega^2A^2 - \frac{1}{2}\frac{q^2E^2}{k}$$

Solution

When electric field is applied, at equilibrium

$F = kx = qE$

$x = \dfrac{qE}{k}$, x- extension of spring

Total Energy of system = Kinetic energy +Potential energy

$T.E. = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

$= \dfrac{1}{2}\dfrac{q^2E^2}{k} + \dfrac{1}{2}m\omega^2A^2$

#868218

In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of $0.25cm$. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is:

A $0.0430cm$

B $0.3150cm$

C $0.4300cm$

D $0.2150cm$

Solution

In one rotation scale moves

$\dfrac{0.25}{5} = 0.05cm$

Least count = $0.05 \times 10^{-2}cm$

For 4 main scale division =$4 \times 0.05 = 0.2cm$

For circular scale divosion = $30 \times 0.05 \times 10^{-2} = 1.5 \times 10^{-2}cm$

Thickness of wire = $0.2 + 0.015 = 0.2150cm$

#868221

A solution containing active cobalt $^{60}_{27}Co$ having activity of $0.8\mu Ci$ and decay constant λ is injected in an animal's body. If $1cm^3$ of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body? ($1Ci = 3.7 \times 10^{10}$ decay per second and at $t = 10hrs$ $e^{-\lambda t} = 0.84$)

A   6 litres

B   7 litres

C   4 litres

D   5 litres

Solution

The activity equation can be written as

$$-\frac{dN}{dt} = \lambda N_o e^{-\lambda t}$$

given that

$$\lambda N_o = 0.8 \mu C_i$$

Putting the values,

$$\lambda N_o = 2.96 \times 10^4$$

Let the volume of the blood flowing be V,

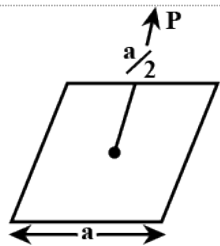the activity would reduce by a factor of $\dfrac{10^{-3}}{V}$

Hence

$$\frac{\lambda N_o 10^{-3}}{V} e^{-\lambda t} = 300/60 \text{ (Both R.H.S. and L.H.S. are decay/s)}$$

Putting the values of $e^{-\lambda t}$ and λN_o we get

$$V = 5 litre$$

#868223

A charge Q is placed at a distance $a/2$ above the centre of the square surface of edge a as shown in the figure.

The electric flux through the square surface is:

A $\dfrac{Q}{3\epsilon_0}$

B $\dfrac{Q}{6\epsilon_0}$

C $\dfrac{Q}{2\epsilon_0}$

D $\dfrac{Q}{\epsilon_0}$

Solution

Draw an imaginary square enclosing that point charge at the centre.

Then from gauss law, $\phi = \dfrac{Q_{in}}{\epsilon_o}$

This is the flux passing through all the six surfaces.

Therefore through one surface, $\dfrac{1}{6}$ th of this flux will pass.

#868224

The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is:

A $20 eV$

B $79 eV$

C $109eV$

D $34eV$

Solution

Let

E_1= energy required to remove

e^-

from singly ionised Helium atom

$$=\frac{+13.6z^2}{n^2}$$

$$=\frac{(13.6)(2)^2}{(1)^2}$$

$$=54.4eV$$

E_2= energy required to remove an e^- from He -atom

given $E_1 = 2.2E_2$

$$\therefore E_2 = \frac{E_1}{2.2} = \frac{54.4}{2.2} = 24.7eV$$

$\therefore$ Energy required to remove both e^-'s from He-atom

$= 24.7 + 54.4$

$= 79.1eV$

#868234

A monochromatic beam of light has a frequency $v = \dfrac{3}{2\pi} \times 10^{12}Hz$ and is propagating along the direction $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for

the magnetic field is:

A $k\dfrac{E_0}{C}\left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right) \quad \cos\left[10^4\left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right).\vec{r}-\left(3\times10^{12}\right)t\right]$

B $\dfrac{E_0}{C}\left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right) \quad \cos\left[10^4\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r}-\left(3\times10^{12}\right)t\right]$

C $\dfrac{E_0}{C}\hat{k} \quad \cos\left[10^4\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r}+\left(3\times10^{12}\right)t\right]$

D $\dfrac{E_0}{C}\dfrac{\left(\hat{i}+\hat{j}+k\right)}{\sqrt{3}} \quad \cos\left[10^4\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r}+\left(3\times10^{12}\right)t\right]$

Solution

Direction of B is ,

$= \hat{K} \times \hat{E}$

$= (\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}) \times \hat{K}$

$= \dfrac{\hat{i}+\hat{k}}{\sqrt{2}} + \dfrac{\hat{j}\times\hat{k}}{\sqrt{2}}$

$= \dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{i}}{\sqrt{2}} = \dfrac{\hat{i}-\hat{j}}{\sqrt{2}}$

So, ans is between (i) and (ii)

Propagation direction , $\hat{k}= \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\bar{B}$ wave eq^n will be

$\Rightarrow \dfrac{E_0}{c}(\hat{B}) \, Cos[|k|\hat{k} - wt]$

$\qquad \downarrow \qquad\qquad \downarrow$

we know it is   we know it is

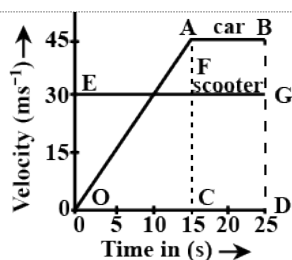$\hat{-} \; \hat{} \qquad\qquad \hat{+} \; \hat{}$

$$\frac{\hat{i} - \hat{j}}{\sqrt{2}} \qquad \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

Only (i) satisfies

Hence , correct answer is (i)

#868241

The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in $15s$ and (ii) the time at which the car will catch up with the scooter are, respectively

A $337.5m$ and $25s$

B $225.5m$ and $10s$

C $112.5m$ and $22.5s$

D $11.2.5m$ and $15s$

Solution

The distance traveled in 15 seconds by both will be given by area under curve.

For car, $s_1 = \dfrac{1}{2} \times 15 \times 45$

$\qquad s_1 = 337.5m$

For scooter, $s_2 = 15 \times 30$

$\qquad\qquad s_2 = 450$

$s_1 - s_2 = 112.5m$

Suppose after time t, they will meet, then distance traveled by both of them will be equal.
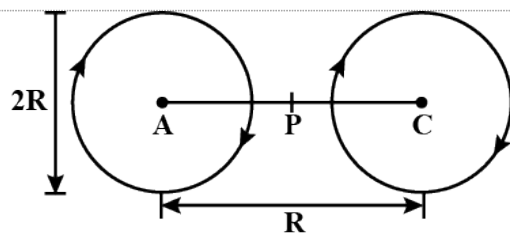
$30t = \dfrac{1}{2} \times 15 \times 45 + 45 \times (t - 15)$

$30t = 337.5 + 45t - 675$

$15t = 337.5$

$t = 22.5s$

#868245

A Helmholtz coil has pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by (Refer to figure):

A $\dfrac{4N\mu_0 I}{5^{3/2}R}$

$\qquad 8N\ _0I$

B	$\dfrac{8N\mu_0 I}{5^{3/2}R}$
C	$\dfrac{4N\mu_0 I}{5^{1/2}R}$
D	$\dfrac{8N\mu_0 I}{5^{1/2}R}$

Solution

The magnetic field due to both the coils are in the same direction and equal in magnitude

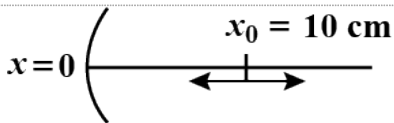The magnitude of the magnetic field due to one coil is give as

$$B = \frac{\mu_o i R^2}{2(R^2 + (R/2)^2)^{3/2}}$$
$$B = \frac{4\mu_o i R^2}{5R^2}$$
$$B_{net} = 2B$$
$$B_{net} = \frac{8\mu_o i R^2}{5R^2}$$

#868255

$$x_0 = 10 \text{ cm}$$
$$x = 0$$

A particle is oscillating on the X-axis with an amplitude $2cm$ about the point $x_0 = 10cm$ with a frequency ω. A concave mirror of focal length $5cm$ is placed at the origin (see figure)

Identify the correct statements:

(A) The image executes periodic motion

(B) The image executes no n-periodic motion

(C) The turning points of the image are asymmetric w.r.t the image of the point at $x = 10cm$

(D) The distance between the turning points of the oscillation of the image is $\dfrac{100}{21}$

A	(B), (D)
B	(B), (C)
C	(A), (C), (D)
D	(A), (D)

Solution

For mean ,
$$\Rightarrow \frac{-1}{10} + \frac{1}{v} = -\frac{1}{5}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{5}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{5}\left[\frac{1}{2} - 1\right]$$

$$\Rightarrow \frac{1}{v} = \frac{-1}{10}$$

$$\Rightarrow v = -10cm$$

As image copies the time period of object (A) is right as well. It will be periodic motion.

For one extreme
$$\Rightarrow \frac{-1}{8} + \frac{1}{v} = -\frac{1}{5}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{8} - \frac{1}{5}$$

$$\Rightarrow \frac{1}{v} = -\frac{3}{40}$$

$$Rightarrow v = \frac{-40}{3} cm$$

For other extreme

$$\Rightarrow \frac{-1}{12} + \frac{1}{v} = -\frac{1}{5}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{12} - \frac{1}{5}$$

$$\Rightarrow \frac{1}{v} = \frac{-7}{60} cm^{-1}$$

$$\Rightarrow v = \frac{-60}{7} cm$$

These points are asymmetric about $x_0 = 10 cm$ So,(c) is right.
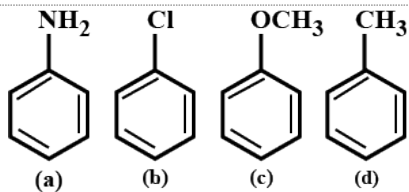
Amplitude of oscillation of image

$$\Rightarrow \frac{40}{3} - \frac{60}{7}$$

$$\Rightarrow 10\left[\frac{4}{3} - \frac{6}{7}\right]$$

$$10 \times \frac{10}{21}$$

$$\Rightarrow \frac{100}{21} cm \text{ (D) is right}$$

#868103

The increasing order of nitration of the following compounds is?

A $(b) < (a) < (c) < (d)$

B $(b) < (a) < (d) < (c)$

C $(a) < (b) < (c) < (d)$

D $(a) < (b) < (d) < (c)$

Solution

Nitration is electrophilic aromatic substitution reaction. Methoxy and amino groups are strongly activating groups. Methyl group is weakly activating group.

Since among methyl and methoxy group, methoxy group is more reactive than methyl group, (c) is more reactive than (d).

Even-though amino group is strongly activating group, it gets protonated in presence of acid) to form anilinium ion which is strongly deactivating. Hence, (a) is less reactive than (c) and (d).

 Chloro group is deactivating group. so (b) has least reactivity.

Note:

The activating groups increases the electron density on benzene ring and increases the rate of electrophilic aromatic substitution reaction. The deactivating groups decreases the electron density on benzene ring and decreases the rate of electrophilic aromatic substitution reaction.

#868105

In graphite and diamond, the percentage of p-characters of the hybrid orbitals in hybridisation are respectively _____.

A 50 and 75

B 67 and 75

C 33 and 25

D 33 and 75

Solution

% p-characters

$= \dfrac{\text{p-orbitals}}{\text{total orbitals}}$

sp^2 in graphite

sp^3 diamond

$\therefore$ for Graphit, % p-characters $= \dfrac{2}{3} \times 100$

$= 67\%$

For diamond, % p-characters $= \dfrac{3}{4} \times 100$

$= 75\%$

#868107

Identify the pair in which the geometry of the species is T-shape and square pyramidal, respectively are _____.

A ClF_3 and IO_4^-

B ICl_2^- and ICl_5

C IO_3^- and $IO_2F_2^-$

D $XeOF_2$ and $XeOF_4$
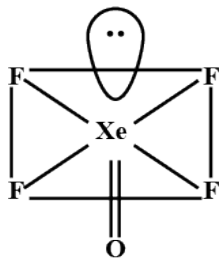
Solution

According, to VSEPR theory

Xe (in

$XeOF_2$) has fine electron pairs with three bonding pair. This means its geometry is T-Shaped. Similarly,

Xe in

$XeOF_4$ has a hybridizah on of

sp^3d^2.

Thus its geometry is square-pyramidal

#868109

N_2O_5 decomposes to NO_2 and O_2 and follows first order kinetics. After 50 minutes, the pressure inside the vessel increases from 50 mm Hg of 87.5 mm Hg. The pressure of the gaseous mixture after 100 minute at constant temperature will be _____.

A   116.25 mmHg

B   106.25 mmHg

C   136.25 mmHg

D   175.0 mmHg

Solution

$N_2O_5 \rightarrow 2NO_2 + \dfrac{1}{2}O_2$
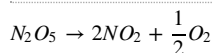
$R = k[N_2O_5]^1$

1. P_o O O

2. $P_o - p$ $2p$ $\dfrac{1}{2}P$ $\rightarrow$ $50min$

3. $P_o - p^1$ $2p^1$ $\dfrac{1}{2}P^1$ $\rightarrow$ $100min$

$P_o = 50$

$\therefore P_{50min} = P_o + \dfrac{3}{2}P = 87.5$

$P = \dfrac{2}{3}(87.5 - 50)$

$P = \dfrac{37.5}{3} \times 2 = 25$

First order Eq^n : $t = \dfrac{2.303}{k}\log\left[\dfrac{(N_2O_5)}{(N_2O_5)_t}\right]$

At 50 min : $t = \dfrac{2.303}{k}\log$

$k = \dfrac{2.303}{50} \times 0.3010$

At 100min : $100 = \dfrac{2.303 \times 50}{2.303 \times 0.3010}\log\left[\dfrac{(N_2O_5)_o}{(N_2O_5)_{100}}\right]$

$2 \times 0.3010 = \log_{10}\left[\dfrac{50}{x}\right]$

$4 = \dfrac{50}{x}$

$x = \dfrac{5025}{4}$

$\therefore P_o - P^i = 12.5$

$P^1 = 50 - 12.5$

$P^1 = 37.5$

$\therefore$ Total Pressure $= P^o + \dfrac{3}{2} \times 37.5$

$106.25 \; mmHg$

#868113

Which of the following statements about colloids is False?

A   Colloidal particles can pass through ordinary filter paper

B   Freezing point of colloidal solution is lower than true solution at same concentration of a solute

C   When silver nitrate solution is added to potassium iodide solution, a negatively charged colloidal solution is formed

D   When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated

Solution

(A) Colloidal particles are so small that they can pass through ordinary filter paper. Also, they cannot be seen with ordinary microscope.

(B) Freezing point of colloidal solution is same as that of true solution at same concentration of a solute. The depression in freezing point is colloidal property and depends on the number of solute particles. It is independent of size or shape of solute particles.

(C) When silver nitrate solution is added to potassium iodide solution, a negatively charged colloidal solution is formed. AgI sol is formed which adsorbs negatively charged iodide ions preferentially. However if the order of addition is reversed, i.e, potassium iodide solution is added to silver nitrate solution, the AgI sol will adsorb positively charged silver ions preferentially.

(D) When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated.Although electrolytes in minute quantities are necessary for the stability of colloids, they cause coagulation of disperse phase if present in large quantities.

#868115
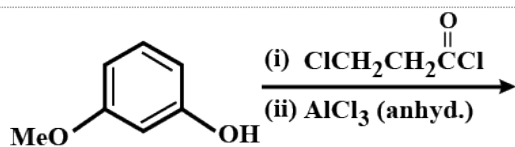

The major product of the following reaction is?

Solution

The major product of the reaction is given by the option (B).

In the first step, acid chloride is more reactive than alkyl halide. Hence, -COCl group will react first.

In the second step, Friedel Craft's alkylation occurs in a position that is ortho to alkoxy group and para to methoxy group. Both methoxy and alkoxy groups are ortho para directing groups.
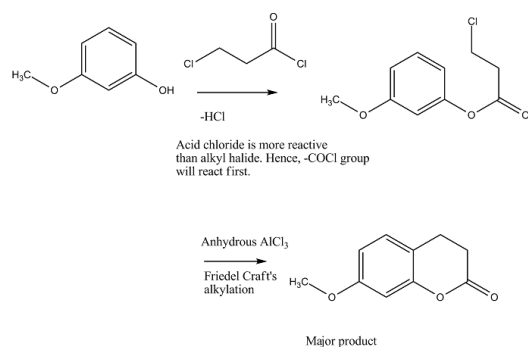
#868120
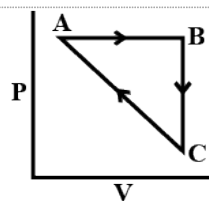
An ideal gas undergoes a cyclic process as shown in Figure.

$\Delta U_{BC} = -5$ mJ mol^{-1}, $q_{AB} = 2$ kJ mol^{-1}

$W_{AB} = -5$ kJ mol^{-1}, $W_{CA} = 3$ kJ mol^{-1}

Heat absorbed by the system during process CA is:

A 18 kJ mol^{-1}

B -18 kJ mol^{-1}

C -5 kJ mol^{-1}

D $+5$ kJ mol^{-1}

Solution

$\Delta U_{AB} = q_{AB} + W_{AB} = 2 + (-5) = -3kJ/mol$

$\Delta U_{BC} = -5kJ/mol$

For cyclic process, $\Delta U = 0$

$\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$

$\Delta U_{CA} = -\Delta U_{AB} - \Delta U_{BC}$

$\Delta U_{CA} = -(-3) - (-5) = 8kJ/mol$

$\Delta U_{CA} = q_{CA} + W_{CA}$

$8 = q_{CA} + 3$
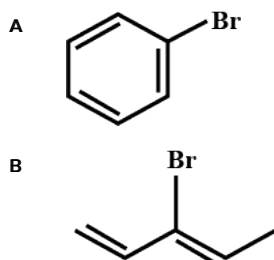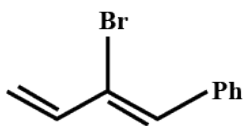
$q_{CA} = +5kJ/mol$

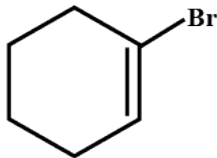Heat absorbed has positive sign.

#868122

Which of the following will most readily give the dehydrohalogenation product?
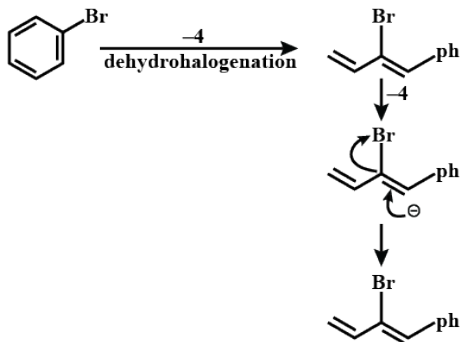
A

B

Solution

Carbanion is adjacent to phenyl ring, so get stabilised by resonance. Hence, the option is C is correct.

#868125

The minimum volume of water required to dissolve 0.1g lead(II) chloride to get a saturated solution (K_{sp} of $PbCl_2 = 3.2 \times 10^{-8}$; atomic mass of Pb$= 207u$) is?
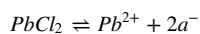
A 0.18 L

B 17.98 L

C 1.798 L

D 0.36 L

Solution

$Ksp = 3.2 \times 10^{-8}$

$PbCl_2 \rightleftharpoons Pb^{2+} + 2a^-$

$O \leq 52$

$K_{sp}(5)(25)^2$

$K_{sp} = 45^3$

$s^3 = \dfrac{3.2 \times 10^{-8}}{4}$

$s^3 = -8 \times 10^{-9}$

$s = 2 \times 10^{-3}$

$m.\,wtg = 207 + 35.5 \times 2$

$PbCl_2 = 278$

$\therefore 2 \times 10^{-3} = \dfrac{\dfrac{0.1}{278}}{x}$

$$\therefore x = \frac{0.1}{278 \times 2 \times 10^{-3}}$$

$$= \frac{100}{278 \times 2}$$

$$x = 0.18L$$

#868129

The decreasing order of bond angles in BF_3, NH_3, PF_3 and I_3^- is?

A $BF_3 > I_3^- > PF_3 > NH_3$

B $I_3^- > NH_3 > PF_3 > BF_3$

C $BF_3 > NH_3 > PF_3 > I_3^-$

D $I_3^- > BF_3 > NH_3 > PF_3$

Solution

$lp - lp > l_p - bp > bp - bp$

Due to $Dp - lp$ repulsion in $PF_3, F - P - F$ cmgle will be less than that of BF_3
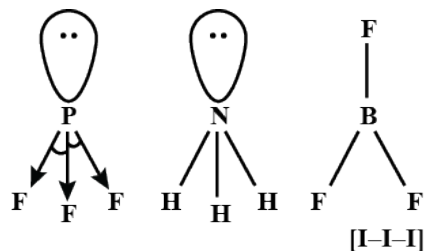
$\therefore BF_3 < PF_3$

$BF_3 > NH_3$, as the geometry of BF_3 is triagonal planar while that NH_3 is tetrahedral

As the Fluorine atom pulls the lone pair of electron on P atom, $lp - bp$ repulsion is more pre dominant. Thus $NH_3 > PF_3$

thus, $BF_3 > NH_3 > PF_3$

I_3^- ion linear. This bond angle is 180°.

$\therefore$ is $I_3^- > BF_3 > NH_3 > PF_3$

#868132

In the molecular orbital diagram for the molecular ion, N_2^+, the number of electrons in the σ_{2p} molecular orbital is?

A   1

B   2
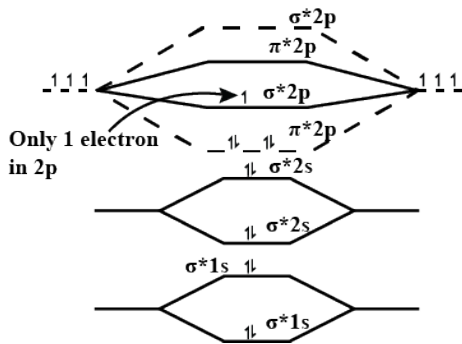
C   0

D   3

Solution

Total electrons

$= 2 \times 7 - 1 = 13$

$\therefore$ Number of electrons in $\sigma_{2p} = 1$

σ*2p
π*2p
σ*2p
π*2p
σ*2s
σ*2s
σ*1s
σ*1s

Only 1 electron in 2p

#868135

The correct combination is?

A $[Ni(CN)_4]^{2-}$ -tetrahedral; $[Ni(CO)_4]$ -paramagnetic

B $[NiCl_4]^{2-}$ -paramagnetic; $[Ni(CO)_4]$ -tetrahedral

C $[NiCl_4]^{2-}$ -diamagnetic; $[Ni(CO)_4]$ -square-planar

D $[NiCl_4]^{2-}$ -square-planar; $[Ni(CN)_4]^{2-}$ -paramagnetic

Solution

$[Ni(CN)_4]^{2-}$ is square planar, diamagnetic (0 unpaired electrons) with dsp^2 hybridisation.

$[Ni(CO)_4]$- is tetrahedral,diamagnetic (0 unpaired electrons)with sp^3 hybridisation.

$[NiCl_4]^{2-}$ is tetrahedral,paramagnetic (2 unpaired electrons) with sp^3 hybridisation.

Hence, the option (B) is the correct answer.

#868137

Ejection of the photoelectron from metal in the photoelectric effect experiment can be stopped by applying 0.5V when the radiation of 250 nm is used. The work function of the metal is?

A 5.5 eV

B 4 eV

C 5 eV

D 4.5 eV

Solution

From Ejection photoelectron

$\because h\nu = h\nu_o + K.E.$

$\dfrac{h\nu}{x} = \phi + \text{Stopping pot}$

$\therefore \phi = \dfrac{hc}{x} - S.P.$

$= \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8}{250 \times 10^{-9} \times 1.6 \times 10^{-19}} - 0.5 \times 1.6 \times 10^{-10}$

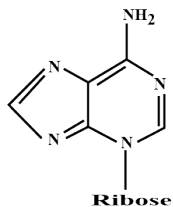$= \dfrac{6.6 \times 3}{250 \times 1.6} \times 10^2 - 0.5$
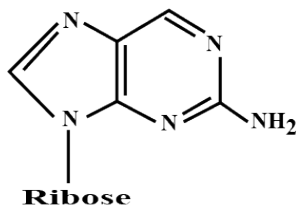
$= 4.95 - 0.5$

$\sim 4.5 eV$

#868138

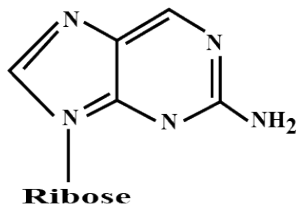Which of the following is the correct structure of Adenosine?

A

B

Ribose

C

Ribose

D

Ribose

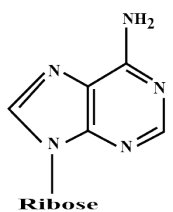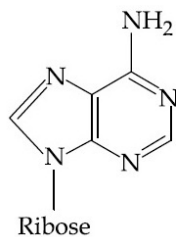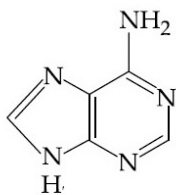**Solution**

The structure of option D is the correct structure of Adenosine. Adenosine is a nucleoside it is a base-sugar unit. Base is adenine and sugar is ribose.

Adenosine

Adenine

#868140

When an electric current is passes through acidified water, 112 mL of hydrogen gas at N.T.P. was collected at the cathode in 965 seconds. The current passed, in ampere, is

_____.

A 0.1

B 2.0

C 1.0

D 0.5

Solution

Reduction at cathode: $2H^+(aq) + 2e^- \rightarrow H_2(g)$

At N.T.P, 22.4 L (or 22400 mL) of $H_2 = 1$ mole of H_2

112 mL of $H_2 = \dfrac{112}{22400} \times 1 = 0.005$ mole of H_2

Moles of H_2 produced $= \dfrac{I(A) \times t(s)}{96500(C/mole^-)} \times$ mole ratio

$0.005 mol = \dfrac{I(A) \times 965s}{96500(C/mole^-)} \times \dfrac{1 \text{ mol } H_2}{2 \text{ mol } e^-}$

$I = 1\ A$

#868141

A sample of $NaClO_3$ is converted by heat to NaCl with a loss of 0.16g of oxygen. The residue is dissolved in water and precipitated as AgCl. The mass of AgCl (in g) obtained will be: (Given: Molar mass of AgCl= 143.5g mol^{-1})

A   0.41

B   0.35

C   0.48

D   0.54

Solution

The molar mass of $O_2 = 32$ g/mol

0.16g of oxygen = $\dfrac{0.16g}{32g/mol} = 0.005$ mol

$2NaClO_3 = 2NaCl + 3O_2$
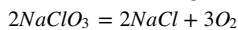
3 moles of $O_2 = 2$ moles of $NaCl = 2$ moles of $AgCl$.

0.005 moles of $O_2 = 0.005 \times \dfrac{2}{3} = 0.003333$ moles of $AgCl$.

Molar mass of AgCl= 143.5g mol^{-1}

The mass of AgCl (in g) obtained will be

$= 143.5gmol^{-1} \times 0.003333mol = 0.48$ g.

#868143
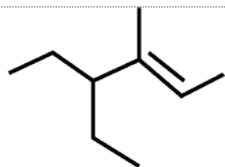
The IUPAC name of the following compound is?

A   3-ethyl-4-methylhex-4-ene

B   4, 4-diethyl-3-methylbut-2-ene

C   4-ethyl-3-methylhex-2-ene

D   4-methyl-3-ethylhex-4-ene

Solution

The IUPAC name of the following compound is $4-ethyl-3methylhex-2-ene$.

#868145

For which of the following reactions, ΔH is equal to ΔU?

A $2HI(g) \rightarrow H_2(g) + I_2(g)$

B $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

C $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

D $2NHO_2(g) \rightarrow N_2O_4(g)$
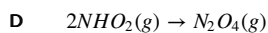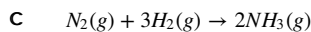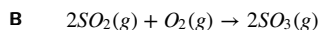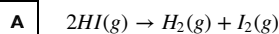
Solution

For the reaction, $2HI(g) \rightarrow H_2(g) + I_2(g)$, ΔH is equal to ΔU

$\Delta H = \Delta U + \Delta nRT$

For the reaction, $2HI(g) \rightarrow H_2(g) + I_2(g)$, $\Delta n = 1 + 1 - 2 = 0$

Hence, $\Delta H = \Delta U + (0)RT$

$\Delta H = \Delta U$

Note: Δn is the difference between the number of moles of gaseous products and the number of moles of gaseous reactants.

(I)   (II)

H - N - - -N - - -N

In hydrogen azide, the bond orders of bonds (I) and (II) are _____.

A   (I)- < 2, (II)- > 2

B   (I)- > 2, (II)- > 2

C   (I)- > 2, (II)- < 2

D   (I)- < 2, (II) < 2

Solution

As in the resonance structure of hydrogen azide, it can be seen than number of bond is

≤ 2 for bond

(I)

Hence its bond order will be < 2

Whereas for bond (II), number of bond ≥ 2

Thus its bond order will be > 2.

Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X' and 'Y' and the oxidation state of Xe are respectively _____.
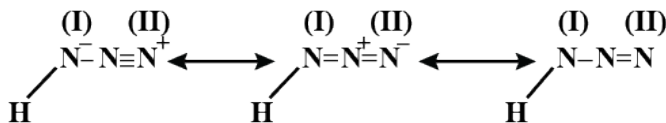
A $XeOF_4(+6)$ and $XeO_2F_2(+6)$

B $XeOF_4(+6)$ and $XeO_3(+6)$

C $XeO_2F_2(+6)$ and $XeO_2(+4)$

D $XeO_2(+4)$ and $XeO_3(+6)$

Solution

Xenon hexafluoride on partial hydrolysis produces compounds $XeOF_4$ and XeO_2F_2. In the compounds $XeOF_4$ and XeO_2F_2, the oxidation states of Xe are $+6$ and $+6$

respectively.
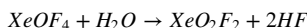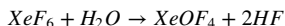
$XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

$XeOF_4 + H_2O \rightarrow XeO_2F_2 + 2HF$

A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aforementioned solution, a white

precipitate is obtained which does not dissolve in dil. nitric acid. The anion is _____.

A CO_3^{2-}

B S^{2-}

C SO_4^{2-}

D Cl^-

Solution

A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aforementioned solution, a white

precipitate is obtained which does not dissolve in dil. nitric acid. The anion is Cl^-.

Cl^- ions react with $AgNO_3$ to form a white curdy ppt of $AgCl$, which is water insoluble and also insoluble in dil HNO_3 but soluble in dilute ammonia due to formation of soluble

complex.

$$Cl^- + Ag^+ \rightarrow AgCl \downarrow$$
$$AgCl + 2NH_3 \rightarrow [Ag(NH_3)_2]Cl$$
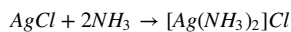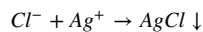
#868153

The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is?

A

$$\left[\begin{array}{c} \overset{\displaystyle CN}{|} \\ CH-CH_2-CH_2-CH \\ | \\ C_6H_5 \end{array} \right]_n$$

B

$$\left[\begin{array}{c} C_6H_5 \quad CN \\ | \qquad | \\ C \; - \; CH-CH_2 \\ | \\ CH_3 \end{array} \right]_n$$

C

$$\left[\begin{array}{c} CH_2-CH-CH_2-CH \\ | \qquad \qquad | \\ C_6H_5 \qquad CN \end{array} \right]_n$$

D

$$\left[\begin{array}{c} C_6H_5 \quad CN \\ | \qquad | \\ CH_2-CH-CH-CH_2 \end{array} \right]_n$$
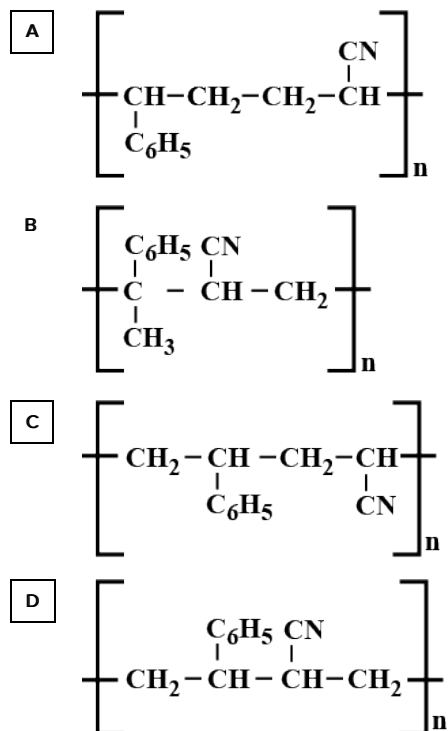
Solution

The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is represented by options (A), (C) and (D)

Styrene is $C_6H_5 - CH = CH_2$ and acrylonitrile is $CH_2 = CH - CN$

$$n \; [\text{styrene}] \quad + \; n \; CH_2 = CH_2 - C \equiv N$$
$$\downarrow H_2O_2$$
$$\left[\begin{array}{c} CH_2 - CH - CH_2 - CH \\ | \qquad \qquad | \\ C_6H_3 \qquad CN \end{array} \right]_n$$
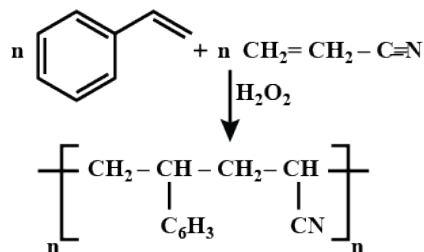
#868158

For Na^+, Mg^{2+}, F^- and O^{2-}; the correct order of increasing ionic radii is?

A $\quad O^{2-} < F^- < Na^+ < Mg^{2+}$

B $\quad Mg^{2+} < Na^+ < F^- < O^{2-}$

C $\quad Mg^{2+} < O^{2-} < Na^+ < F^-$

D $\quad Na^+ < Mg^{2+} < F^- < O^{2-}$

Solution

All the species are iso-electronic

$(10e^-)$

For iso-electronic species more the positive change, smaller the ionic radii

For iso-electronic species more negative change, bigger the ionic radii

Hence the order is $Mg^{2+} < Na^+ < F^- < O^{2-}$

#868164

In which of the following reactions, an increase in the volume of the container will favour the formation of products?

| A | $4NH_3(g) + 5O_2(g) \rightleftharpoons 4NO(g) + 6H_2O(l)$ |

| B | $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$ |

| C | $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ |

| D | $3O_2(g) \rightleftharpoons 2O_3(g)$ |
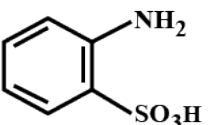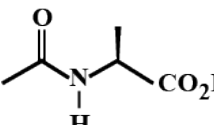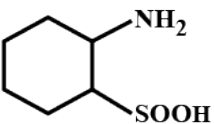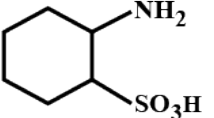
Solution

Increase in volume will favour formation of product, if number of moles of reactant is greater than product.

In $3O_2 \rightleftharpoons 2O_3(g)$, the condition is satisfied. Hence, $3O_2 \rightleftharpoons 2O_3$ is the answer.

#868167

Which of the following will not exist in zwitter ionic form at pH= 7?

A

B

C

D

Solution

The compound in option (B) will not exist in zwitter ionic form at pH=7 as N is in the form of amide group and not in the form of amine. Hence, at pH=7, amide N will not be protonated.

#868173
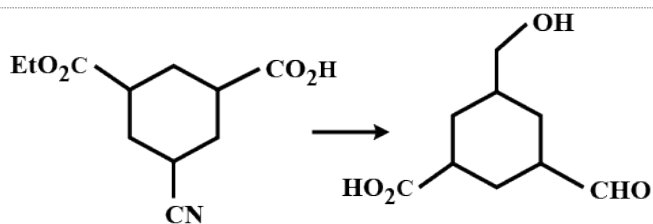
The reagent(s) required for the following conversion are _____.

A (i) $NaBH_4$, (ii) Raney Ni/H_2 (iii) H_3O^+

B (i) B_2H_6 (ii) $SnCl_2/HCl$ (iii) H_3O^+

C (i) $LiAlH_4$ (ii) H_3O^+

| D | (i) B_2H_6 (ii) DIBAL-H (iii) H_3O^+ |

Solution

The reagent required for the following reaction is , B_2H_6, $DIBAL - H$, H_3O^+.
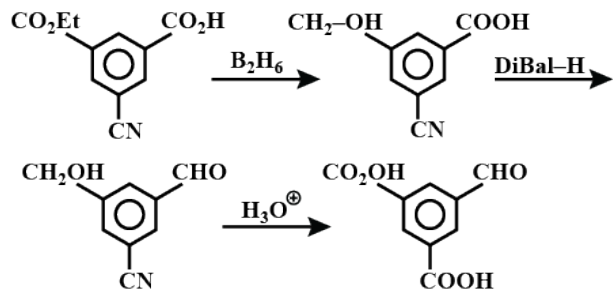
#868179

The correct match between items of List-I and List-II is?

List-I	List-II
(A) Coloured impurity	(P) Steam distillation
(B) Mixture of o-nitrophenol and p-nitrophenol	(Q) Fractical distillation
(C) Crude Naphtha	(R) Charcoal treatment
(D) Mixture of glycerol and sugars	(S) Distillation under reduced pressure

A   (A)-(R), (B)-(S), (C)-(P), (D)-(Q)

B   (A)-(R), (B)-(P), (C)-(S), (D)-(Q)

C   (A)-(P), (B)-(S), (C)-(R), (D)-(Q)

D   (A)-(R), (B)-(P), (C)-(Q), (D)-(S)

Solution

(A) Charcoal treatment removes coloured impurity through adsorption.

(B) Steam distillation separates the

mixture of o-nitrophenol and p-nitrophenol. The o-nitrophenol is steam volatile (due to intramolecular hydrogen bonding), and the para isomer is not volatile.

(C) Fractional distillation separates Crude Naphtha. Naphtha is a flammable liquid hydrocarbon mixture.

(D) Distillation under reduced pressure separates mixture of glycerol and sugars. Vacuum distillation lowers the boiling point and prevents decomposition.

#868181

Which of the following is a Lewis acid?

A $B(CH_3)_3$

B   NaH

C NF_3

D PH_3

Solution
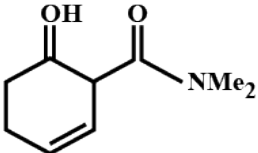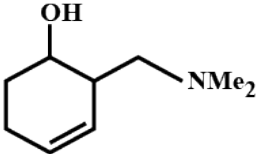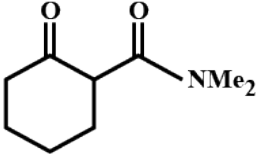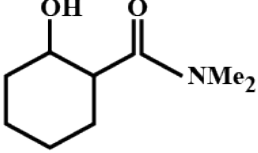
(ref. image) is a Lewis acid. It has empty orbital to accept electron pair.
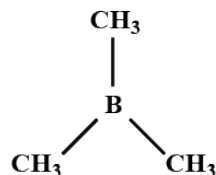
#868184
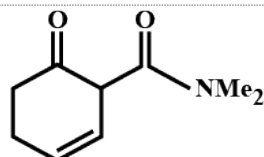
The main reduction product of the following compound with $NaBH_4$ in methanol is?

A

OH   O
(structure with NMe₂)

B

OH
(structure with NMe₂)

C

O   O
(structure with NMe₂)

D

OH   O
(structure with NMe₂)

Solution

Sodium borohydride will reduce ketone to alcohol. It will not reduce amide group and $C = C$ double bond. Hence, option (A) is the correct answer.

Sodium
borohydride

OH   O

Reduction
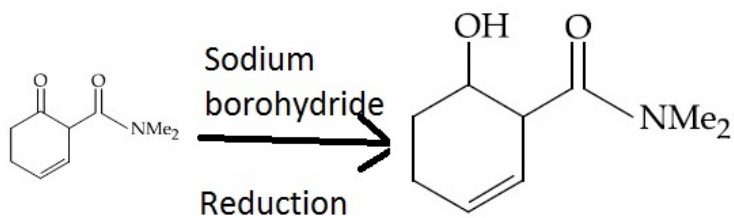
#868185

Which of the following arrangements shows the schematic alignment of magnetic moments of antiferromagnetic substance?

A ↑ ↑ ↑ ↑ ↑ ↑

B ↑ ↑ ↓ ↑ ↑ ↓

C ↑ ↓ ↓ ↓ ↓ ↑

D ↑ ↓ ↑ ↓ ↑ ↓

Solution

The arrangement of option (D) shows the schematic alignment of magnetic moments of anti-ferromagnetic substance.

In an anti-ferromagnetic substance, the magnetic moments align in a regular pattern with neighboring spins pointing in opposite directions.

In a triangle ABC, coordinates of A are $(1, 2)$ and the equations of the medians through B and C are $x + y = 5$ and $x = 4$ respectively,Then area of $\triangle ABC$ (in sq. units) is

A   5

B   9

C   12

D   4

Solution

Median through

C is

$x = 4$

So clearly the x coordinate of C is 4. So let $C = (4, y)$, then the midpoint of $A(1, 2)$ and $C(2, y)$ which is D lies on the median through B by definition. Clearly,

$D = (\dfrac{1 + 4}{2}, \dfrac{2 + y}{2})$.

Now, we have, $\dfrac{3 + 4 + y}{2} = 5 \Rightarrow y = 3$. So, $C = (4, 3)$.

The centroid of the triangle is the intersection of the medians. It is easy to see that the medians $x = 4$ and $x + y = 5$ intersect at $G = (4, 1)$.

The area of triangle

$\triangle ABC = 3 \times \triangle AGC = 3 \times \dfrac{1}{2} \times 3 \times 2 = 9$.

(In this case, it is easy as the points G and C lie on the same vertical line $x = 4$. So the base $GC = 2$ and the altitude from A is 3 units.)

So the answer is option B.

If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval.

A $(-\infty, -10]$

B $(10, \infty)$

C $(0, 10)$

D $(-10, 0)$

Solution

first term

$= b$

common ration $= d$

$-1 < d < 1$ for infinte series

For infinite series ;

$S = \dfrac{b}{1 - d} = 5$

$\rightarrow b = 5(1 - b)$

interval of $b = (0, 10)$

Let

r be the common ration of the series, then the sum of the series is

$\dfrac{b}{1 - r} = 5$. Clearly,

$b = 5(1 - r)$.

Since $|r| < 1$ for an infinite geometric series to converge, we have $-1 < r < 1 \Rightarrow 0 < 1 - r < 2 \Rightarrow 0 < b < 10$.

So option C is the correct answer.

A variable plane passes through a fixed point $(3, 2, 1)$ and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to $yz - plane$ through A, a second plane

is drawn parallel $zx - plane$ through B and a third plane is drawn parallel to $xy - plane$ through C. Then the locus of the point of intersection of these three planes, is

A $x + y + z = 6$

B $\dfrac{x}{3} + \dfrac{y}{2} + \dfrac{z}{1} = 1$

C $\dfrac{3}{x} + \dfrac{2}{y} + \dfrac{1}{z} = 1$

D $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{11}{6}$

Solution

Let

a, b, c be the intercepts of the variable plane on the

x, y, z axes respectively, then the equation of the plane is

$\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$.

And the point of intersection of the planes parallel to the xy, yz, zx planes is clearly the point (a, b, c).

Since the point $(3, 2, 1)$ lies on the variable plane, we have $\dfrac{3}{a} + \dfrac{2}{b} + \dfrac{1}{c} = 1$.

Thus the required locus is $\dfrac{3}{x} + \dfrac{2}{y} + \dfrac{1}{z} = 1$

So the answer is option C.

#868121

If $f\left(\dfrac{x-4}{x+2}\right) = 2x + 1, (x \epsilon R = \{1, -2\})$, then $\int f(x)dx$ is equal to

(where C is a constant of integration)

A $12 \log_e |1 - x| - 3x + c$

B $-12 \log_e |1 - x| - 3x + c$

C $-12 \log_e |1 - x| + 3x + c$

D $12 \log_e |1 - x| + 3x + c$

Solution

Let

$\dfrac{x-4}{x+2} = y \Rightarrow x - 4 = yx + 2y \Rightarrow x(1 - y) = 2y + 4 \Rightarrow x = \dfrac{2y + 4}{1 - y}$

This gives us $f(y) = 2\left(\dfrac{2y + 4}{1 - y}\right) + 1$

So, we have $f(x) = 2\left(\dfrac{2x + 4}{1 - x}\right) + 1 = \dfrac{3x + 9}{1 - x} = -3\left(\dfrac{x - 1 + 4}{x - 1}\right) = -3 - \dfrac{12}{x - 1}$

Thus $\int f(x)dx = 12 \log_e |1 - x| - 3x + c$

So, the correct answer is option A.

#868124

If $\lambda \epsilon R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this

equation is

A   20

B $2\sqrt{5}$

C $2\sqrt{7}$

D $4\sqrt{2}$

Solution

By quadratic formula, the roots of this equation are:

$\alpha, \beta = \dfrac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \dfrac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$.

The magnitude of the difference of the roots is clearly $|\sqrt{\lambda^2 - 36}|$

We have, $\alpha^3 + \beta^3 = \dfrac{(\lambda - 2)^3}{4} + \dfrac{3(\lambda - 2)(\lambda^2 - 36)}{4} = \dfrac{(\lambda - 2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda - 2)(\lambda^2 - \lambda - 26)$.

This function attains its minimum value at $\lambda = 4$.

Thus, the magnitude of the difference of the roots is clearly $|i\sqrt{20}| = 2\sqrt{5}$.

So the correct answer is option B.

#868127

Consider the following two binary relations on the set $A = \{a, b, c\}$: $R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$

and $R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}$ Then

A	R_2 is symmetric but it is not transitive

B   Both R_1 and R_2 are transitive

C   Both R_1 and R_2 are not symmetric

D R_1 is not symmetric but it is transitive

Solution

Both

R_1 and

R_2 are symmetric as for any

$(a_1, a_2) \in R_1$, we have

$(a_2, a_1) \in R_1$ and same thing can be verified for

R_2 as well (

$a_1 \neq a_2$).

For checking transitivity, we observe for R_2 that $(b, a) \in R_2, (a, c) \in R_2$ but $(b, c) \notin R_2$.

Similarly, for R_1, $(b, c) \in R_1, (c, a) \in R_1$ but $(b, a) \notin R_1$. So neither R_1 nor R_2 is transitive.

So, the correct answer is option A.

#868130

If $x^2 + y^2 + \sin y = 4$, then the value of $\dfrac{d^2 y}{dx^2}$ at the point $(-2, 0)$ is

A	-34

B -32

C -2

D 4

Solution

Given,

$x^2 + y^2 + \sin y = 4$

On differentiating the equation the above equation w.r.t x, we get

$$2x + 2y\frac{dy}{dx} + \cos y\frac{dy}{dx} = 0 \dots (i)$$
$$\Rightarrow 2x + (2y + \cos y)\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y + \cos y}$$

at $(-2, 0)$, $\dfrac{dy}{dx} = \dfrac{-2 \times -2}{2 \times 0 + \cos 0}$
$$\Rightarrow \frac{dy}{dx} = \frac{4}{0 + 1}$$
$$\Rightarrow \frac{dy}{dx} = 4 \dots (ii)$$

Again differentiating equation (i) w.r.t to x, we get

$$2 + 2(\frac{dy}{dx})^2 + 2y\frac{d^2 y}{dx^2} - \sin y(\frac{dy}{dx})^2 + \cos y\frac{d^2 y}{dx^2} = 0$$
$$\Rightarrow 2 + (2 - \sin y)(\frac{dy}{dx})^2 + (2y + \cos y)\frac{d^2 y}{dx^2} = 0$$

$$\Rightarrow (2y + \cos y)\frac{d^2y}{dx^2} = -2 - (2 - \sin y)(\frac{dy}{dx})^2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2 - (2 - \sin y)(\frac{dy}{dx})^2}{2y + \cos y}$$

Therefore, at (-2,0),

$$\frac{d^2y}{dx^2} = \frac{-2 - (2 - 0) \times 4^2}{2 \times 0 + 1}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2 - 2 \times 16}{1}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -34$$

Hence, correct option is $(A) - 34$

#868133

A circle passes through the points $(2, 3)$ and $(4, 5)$. If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to

A $\sqrt{5}$

B 1

C $\sqrt{2}$

D 2

Solution

Equation of the line through the given points is

$y - 3 = x - 2 \Rightarrow x - y + 1 = 0$.

Equation of the perpendicular line through the midpoint $(3, 4)$ is $x + y - 7 = 0$.

This intersects the given line at the center of the circle. So, the center of the circle is found to be $(2, 5)$.

Clearly, the radius is then $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 2)^2 + (3 - 5)^2} = 2$ units.

So the answer is option D.

#868134

If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is

A $x^2 - 4y^2 + 16x^2y^2 = 0$

B $4x^2 - y^2 + 16x^2y^2 = 0$

C $4x^2 - y^2 - 16x^2y^2 = 0$

D $x^2 - 4y^2 - 16x^2y^2 = 0$

Solution

$4y^2 = x^2 + 1$

$\Rightarrow -x^2 + 4y^2 = 1$

$$\Rightarrow -\frac{x^2}{1^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

$a = 1, b = \dfrac{1}{2}$

Let, tangent to the curve is at point (x_1, y_1).

$\therefore 4 \times 2y \cdot \dfrac{dy}{dx} = 2x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2x_1}{8y_1} = \dfrac{x_1}{4y_1}$

$\therefore Eq^n$ of tangent: $y = mx + c$

$\Rightarrow y = \dfrac{x_1}{4y_1} \cdot x + c$

$\Rightarrow y_1 = \dfrac{x_1 y_1}{4y_1} + c$

$\Rightarrow c = y_1 = \dfrac{x_1^2}{4y_1}$

$= \dfrac{4y_1^2 - x_1^2}{4y_1} = \dfrac{1}{4y_1}$

$\Rightarrow y = \dfrac{x_1}{4y_1}x + \dfrac{1}{4y_1}$

$\Rightarrow 4y_1 y = x_x x + 1 \ldots (I)$

Intersects x axis at $\left(\dfrac{-1}{x_1}, 0\right)$

And y axis at $= \left(0, \dfrac{1}{4y_1}\right)$

$h = \dfrac{-1}{2x_1}$

$x_1 = \dfrac{-1}{2h}$

$y_1 = \dfrac{1}{8k}$

Midpoint : $\left(\dfrac{-1}{2x_1}, \dfrac{1}{8y_1}\right) : (h, k)$

$4y_1^2 \neq x_1^2 + 1$

$\Rightarrow 4\left(\dfrac{1}{8k}\right)^2 = \left(\dfrac{-1}{2h}\right)^2 + 1$

$\Rightarrow \dfrac{1}{16k^2} = \dfrac{1}{4h^2} + 1$

$\Rightarrow 1 = \dfrac{16k^2}{4h^2} + 16k^2$

$h^2 = 4k^2 + 16h^2 k.$

$x^2 - 4y^2 - 16x^2 y^2 = 0$

This is the required equation.

#868136

Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals.

A $\quad \begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

B $\quad \begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

C $\quad \begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

D $\quad \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

Solution

A is a matrix such that

$A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and

$|3A| = 108$

Let the scalar matrix be $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\Rightarrow A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^{-1}$ ..... $[\because AB = C \Rightarrow A = CB^{-1}]$

Let $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

Now, $|B| = 3$

Then, $B^{-1} = \dfrac{1}{|B|}$ Co-factor matrix of B

$\Rightarrow A = \dfrac{1}{3} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 1 & -\dfrac{2}{3} \\ 0 & \dfrac{1}{3} \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & -\dfrac{2}{3}k \\ 0 & \dfrac{k}{3} \end{bmatrix}$ .... (i)

$|3A| = 108$ .... [Given]

$\Rightarrow 108 = |3A| = 3|A| = \begin{vmatrix} 3k & -2k \\ 0 & k \end{vmatrix}$

$\Rightarrow 3k^2 = 108$

$\Rightarrow k^2 = 36$

$\Rightarrow k = \pm 6$

Take $k = 6$

$\Rightarrow A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$ ...... From (i)

$\Rightarrow A^2 = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

For $k = -6$

$\Rightarrow A = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix}$ ...... From (i)

$\Rightarrow A^2 = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

Hence, $A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

#868155

If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim\limits_{x \to 0} \dfrac{f'(x)}{x}$.

| A | Exists and is equal to -2 |

| B | Does not exist |

C Exist and is equal to 0

D Exists and is equal to 2

Solution

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$= \cos x(x^2 - 2x^2) - x(2\sin x - 2x\tan x) + 1(2x\sin x - x^2\tan x)$

$= -x^2\cos x - 2x\sin x + 2x^2\tan x + 2x\sin x - x^2\tan x$

$= x^2\tan x - x^2\cos x$

$= x^2(\tan x - \cos x)$

$f'(x) = 2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)$

$\therefore \lim\limits_{x\to 0}\dfrac{f'(x)}{x} = \lim\limits_{x\to 0}\dfrac{2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)}{x}$

$= \lim\limits_{x\to 0} 2(\tan x - \cos x) + x(\sec^2 x + \sin x)$

$= 2(0-1) + 0 = -2$

Hence, $\therefore \lim\limits_{x\to 0}\dfrac{f'(x)}{x} = -2$

#868159

If $x_1, x_2, \ldots, x_n$ and $\dfrac{1}{h_1}, \dfrac{1}{h_2}, \ldots, \dfrac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals.

A 2560

B 2650

C 3200

D 1600

Solution

Let

d_1 be the common difference of the A.P.

$x_1, x_2, \ldots,$ then

$x_8 - x_3 = 5d_1 = 12 \Rightarrow d_1 = \dfrac{12}{5} = 2.4$

$\Rightarrow x_5 = x_3 + 2d_1 = 8 + 2 \times \dfrac{12}{5} = 12.8$

Let d_2 be the common difference of the other sequence then $5d_2 = \dfrac{1}{20} - \dfrac{1}{8} = \dfrac{-3}{40} \Rightarrow d_2 = \dfrac{-3}{200}$

$\Rightarrow \dfrac{1}{h_{10}} = \dfrac{1}{h_7} + 3d_2 = \dfrac{1}{200} \Rightarrow h_{10} = 200$

$\Rightarrow x_5 \cdot h_{10} = 12.8 \times 200 = 2560$

So option A is the correct answer.

#868163

The mean of a set of 30 observations is 75. If each other observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. The λ is equal to

A $\dfrac{10}{3}$

B $\dfrac{4}{3}$

C $\dfrac{1}{3}$

D $\dfrac{2}{3}$

Solution

Since mean is a linear operation, after taking product with the same number and subtracting the same number, the mean becomes

$75\lambda - 25$.

We thus have by the condition given in the problem $75\lambda - 25 = 75 \Rightarrow \lambda = \dfrac{4}{3}$.

So the correct answer is option B.

#868166

If n is the degree of the polynomial, $\left[\dfrac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\dfrac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to

A $(12, (20)^4)$

B $(8, 5(10)^4)$

C $(24, (10)^8)$

D $(12, 8(10)^4)$

Solution

$$\left[\dfrac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\dfrac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

Rotionalise the polynomial,

$$\left[\dfrac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \times \dfrac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8 + \left[\dfrac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \times \dfrac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8$$

$$= \left[\dfrac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8 + \left[\dfrac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8$$

$$= \dfrac{1}{2^8}\left[\left[\sqrt{5x^3+1}+\sqrt{5x^3-1}\right]^8 + (\sqrt{5x^3+1}-\sqrt{5x^3-1})^8\right]$$

$$= \dfrac{1}{2^8}\left[(a+b)^8 + (a-b)^8\right]$$

we know, $(a+b)^8 = {}^8C_0\,a^8b^0 + {}^8C_1 a^7 b^1 + \ldots + {}^8C_8 a^0 b^8$
$(a-b)^8 = {}^8C_0 a^8 b^0 - {}^8C_1 a^7 b^1 + \ldots + {}^8C_8 a^0 b^8$
$(a+b)^8 + (a-b)^8 = {}^8C_0 a^8 b^0 + {}^8C_2 a^6 b^2 + {}^8C_4 a^4 b^4 + {}^8C_6 a^2 b^6 + {}^8C_8 a^0 b^8$

Thus, our expression becomes,

$$= \dfrac{1}{2^8}\left[\begin{array}{l}{}^8C_0(\sqrt{5x^3+1})^8 + {}^8C_2(\sqrt{5x^3+1})^6(\sqrt{5x^3-1})^2 + {}^8C_4(\sqrt{5x^3+1})^4(\sqrt{5x^3-1})^4 + \\ {}^8C_6(\sqrt{5x^3+1})^2(\sqrt{5x^3-1})^6 + {}^8C_8(\sqrt{5x^3-1})^8\end{array}\right]$$

$$= \dfrac{1}{2^8}\left[\begin{array}{l}{}^8C_0(5x^3+1)^4 + {}^8C_2(5x^3+1)^3(5x^3-1) + {}^8C_4(5x^3+1)^2(5x^3-1)^2 + \\ {}^8C_6(5x^3+1)(5x^3-1)^3 + {}^8C_8(5x^3-1)^4\end{array}\right]$$

From this, we can clearly see that the degree of polynomial is 12,

hence $h = 12$ which means option (2) & (3) are incorrect.

now, for m, let collect the coefficients of x^{12} from each term.

coefficient of $x^{12} = \left[{}^8C_0 5^4 + {}^8C_2 5^4 + {}^8C_4 5^4 + {}^8C_6 5^4 + {}^8C_8 5^4\right]$

$$= \left[5^4 \times \dfrac{2^8}{2}\right]$$
$$= 5^4 \times 2^4 \times \dfrac{2^4}{2}$$
$$= 10^4 \times 2^3$$
$$= 8(10^4)$$

Hence, option D is correct.

#868169

Let S be the set of all real values of k for which the system of linear equations

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

has a unique solution. Then S is

A   An empty set

B   Equal to $R - \{0\}$

C   Equal to $\{0\}$

D   Equal to R

Solution

Given are the system of linear equations.

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

We know that, this system has a unique solution.

Therefore, coefficient determinant is non-zero.

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$\Rightarrow k + 2 - (2k + 3) + 1 \neq 0$

$\Rightarrow k \neq 0$

Therefore, $k \in R - \{0\} \equiv S$

#868171

The value of the integral $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log\left(\frac{2 + \sin x}{2 - \sin x}\right)\right) dx$ is

A $\dfrac{3}{16}\pi$

B 0

C $\dfrac{3}{8}\pi$

D $\dfrac{3}{4}$

Solution

Let

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log\left(\frac{2 + \sin x}{2 - \sin x}\right)\right) dx \quad \text{------}$$
i

Now we know that, $\int_a^b f(x).\,dx = \int_a^b f(a + b - x).\,dx$

$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^4 (-x)).(1 + \log(\frac{1 + \sin(-x)}{1 - \sin(-x)})).\,dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^4 x).(1 + \log(\frac{1 - sinx}{1 + sinx})).\,dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x.(1 - \log(\frac{1 + sinx}{1 - sinx})).\,dx \quad \text{------}ii$

Adding equation i and ii we get,

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x.\,dx$

$2I = 2\int_0^{\frac{\pi}{2}} \sin^4 x.\,dx$

$I = \int_0^{\frac{\pi}{2}} \sin^4 x.\,dx$

Use the reduction formula

$\int \sin^m x \cdot dx = \frac{-\cos x \cdot \sin^{m-1}(x)}{m} + \frac{m-1}{m} \int \sin^{-2+m} x \cdot dx$

Here, $\int_0^{\frac{\pi}{2}} \sin^4 x \cdot dx = \frac{-\cos x \cdot \sin^3 x}{4}\Big|_0^{\frac{\pi}{2}} + \frac{3}{4} \int_0^{\frac{\pi}{2}} \sin^2 x \cdot dx$ ----------As $\frac{-\cos x \cdot \sin^3 x}{4}\Big|_0^{\frac{\pi}{2}} = 0$

$$\Rightarrow \frac{3}{4} \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} \cdot dx$$

$$= \frac{3}{8} \int_0^{\frac{\pi}{2}} 1 \cdot dx - \frac{3}{8} \int_0^{\frac{\pi}{2}} \cos 2x \cdot dx$$

After putting the limits we get,

$$= \frac{3\pi}{16} \text{ -------As}(\int_0^{\frac{\pi}{2}} \cos 2x \cdot dx) = 0$$

$$\Rightarrow \frac{3\pi}{16}$$

#868174

An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is

A $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$

B $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

C $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$

D $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Solution

$3x + 4y + z = 1 \times 2$

$5x + 8y + 2z = -14$

$6x + 8y + 2z = 2$

$x = 16; 4y + z = 1 - 48$

$4y + z = -47$

$x, 4y + z = -47$

$(15, -12, 1)$ and $(15, -11, -3)$

$x = 15; \frac{y+12}{1} = \frac{z-1}{-4}$ $\vec{r} = (15, 12, 1) + \lambda(0 - 1, -4)$

$x - 15 = 0; \frac{y+12}{1} = z$

$\cos\theta = \frac{\vec{b} \cdot \vec{x}}{|b||n|}$

$= \frac{(0\hat{i} + 1\hat{j} - 4k)(\hat{i} + \hat{jk})}{\sqrt{17} \times 3} \vec{x} = (1, 1, 1)$

$= \frac{1-4}{\sqrt{51}} = \frac{3}{\sqrt{17 \times 3}} = \sqrt{\frac{3}{17}}$

$\theta = \cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$.

Normal to

$3x + 4y + z = 1$ is

$3\hat{i} + 4\hat{j} + \hat{k}$.

Normal to $5x + 8y + 2z = -14$ is $5\hat{i} + 8\hat{j} + 2\hat{k}$

The line at which these planes intersect is perpendicular to both normals, hence its direction ratios are directly proportional to the cross product vector of the normals

So, the direction ratios of the line can be chosen as $-\hat{j} + 4\hat{k}$

So, the angle between the plane $x + y + z + 5 = 0$ and the line obtained is given by $\sin^{-1} \frac{-1+4}{\sqrt{17}\sqrt{3}} = \sin^{-1}\sqrt{\frac{3}{17}}$

So, option D is the correct answer.

If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is

| A | $8\sqrt{3}\pi$ |

B $6\sqrt{2}\pi$

C $6\sqrt{3}\pi$

D $8\sqrt{2}\pi$

Solution

Sphere of radius : 3 $cm(r=3)$

Let b, h be radius and height of sphere, respectively.

$\therefore$ volume of cone $= \dfrac{1}{3}\pi b^2 h$

In $\triangle ABC$, using Pythagoras theorem

$(h-r)^2 + b^2 = r^2 \ldots \ldots (i)$

$b^2 = r^2 - (h-r)^2 = r^2 - (h^2 - 2hr + r^2) = 2hr - h^2$

$\therefore$ Volume $v = \dfrac{1}{3}h[r^2 - (h-r)^2]$

$= \dfrac{1}{3}\pi h[2hr - h^2] = \dfrac{1}{3}[2h^2 r - h^3]$

$\dfrac{dv}{dh} = \dfrac{1}{3}[4hr - 3h^2] = 0 \Rightarrow h(4r - 3h) = 0$

$\dfrac{d^2 v}{dh^2} = \dfrac{1}{3}[4r - 6h]$

At $h = \dfrac{4r}{3}, \dfrac{d^2 v}{dh^2} = \dfrac{1}{3}[4r - \dfrac{4r}{3} \times 6]$

$= \dfrac{1}{3}[4r - 8r] < 0 \Rightarrow$ maximum volume at $h = \dfrac{4r}{3}$

$h = \dfrac{4r}{3} = 4$

$\therefore$ From (1)

$(h-r)^2 + b^2 = r^2$

$\Rightarrow b^2 = 2hr - h^2$

$= 2 \cdot \dfrac{4r}{3}r - \dfrac{16r^2}{9}$

$= \dfrac{8r^2}{3} - \dfrac{16r^2}{9}$

$= \dfrac{(24 - 16)r^2}{9} = \dfrac{8r^2}{9}$

$\Rightarrow b = \dfrac{2\sqrt{2}}{3}r \Rightarrow 2\sqrt{2}$
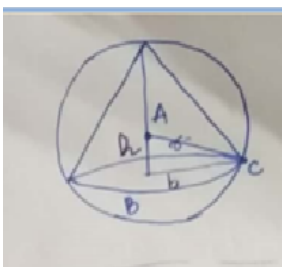
Curved surface area $= \pi bl$

$= \pi b\sqrt{h^2 + r^2}$

$= \pi 2\sqrt{2}\sqrt{4^2 + 8}$

$= \pi 2\sqrt{2}\sqrt{24}$

$= \pi 2\sqrt{2}2\sqrt{3}\sqrt{2}$

$= 8\sqrt{3}\pi$.

If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3\sin^2(A + B) - 10\sin(A + B) \cdot \cos(A + B) - 25\cos^2(A + B)$ is

A   25

B -25

C -10

D   10

Solution

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Using the fact that $\tan A$ and $\tan B$ are the roots of $3x^2 - 10x - 25 = 0$, we get $\tan(A + B) = \dfrac{10/3}{28/3} = \dfrac{5}{14}$

We also see that $\cos 2(A + B) = -1 + 2\cos^2(A + B) = \dfrac{1 - \tan^2(A + B)}{1 + \tan^2(A + B)} \Rightarrow \cos^2(A + B) = \dfrac{196}{221}$

We see that

$3\sin^2(A + B) - 10\sin(A + B)\cos(A + B) - 25\cos^2(A + B) = \cos^2(A + B)(3\tan^2(A + B) - 10\tan(A + B) - 25)$

$= \dfrac{75 - 700 - 4900}{196} \times \dfrac{121}{221} = -\dfrac{5525}{196} \times \dfrac{196}{221} = -25$

So option B is the correct answer.

Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is

3,

then the equation of the common tangent to the two parabolas is?

A $3(x + y) + 4 = 0$

B $8(2x + y) + 3 = 0$

C $4(x + y) + 3 = 0$

D $x + 2y + 3 = 0$

Solution

Origin $(0, 0)$ is the only point common to x-axis and y-axis.

$\Rightarrow$ Origin $(0, 0)$ is the common vertex

Let the equation of 2 parabola be $y^2 = 4ax$ and $x^2 = 4by$

Latus rectum$= 3$

$\Rightarrow 4a = 4b = 3$

$\Rightarrow a = b = \dfrac{3}{4}$

$\therefore$ The 2 parabolas are $y^2 = 3x$ and $x^2 = 3y$

Let $y = mx + c$ be the common tangent

$y^2 = 3x$

$\Rightarrow (mx + c)^2 = 3x$

$\Rightarrow m^2x^2 + (2mc - 3)x + c^2 = 0$

The tangent touches at only one point

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (2mc - 3)^2 - 4m^2c^2 = 0$

$\Rightarrow 4m^2c^2 + 9 - 12mc - 4m^2c^2 = 0$

$\Rightarrow c = \dfrac{9}{12m} = \dfrac{3}{4m}$ ....(1)

$m^2 = -c = \dfrac{-3}{4m}$

$x^2 = 3y$

$\Rightarrow x^2 = 3(mx + c)$

$\Rightarrow x^2 - 3mx - 3c = 0$

Tangent touches at only one point

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow 9m^2 - 4(1)(-3c) = 0$

$\Rightarrow 9m^2 = -12c$ ....(2)

From (1) and (2)

$m^2 = \dfrac{-4c}{3} = \dfrac{-4}{3}\left(\dfrac{3}{4m}\right)$

$\Rightarrow m^3 = -1$

$\Rightarrow m = -1$

$\Rightarrow c = \dfrac{-3}{4}$

$\therefore y = mx + c = -x - \frac{3}{4}$

$\Rightarrow 4(x + y) + 3 = 0$

#868187

Let $y = y(x)$ be the solution of the differential equation $\dfrac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & otherwise \end{cases}$

If $y(0) = 0$, then $y\left(\dfrac{3}{2}\right)$ is

A $\dfrac{e^2 - 1}{2e^3}$

B $\dfrac{e^2 - 1}{e^3}$

C $\dfrac{1}{2e}$

D $\dfrac{e^2 + 1}{2e^4}$

Solution

Solving the initial value problem, we get

$y = \dfrac{1}{2} - \dfrac{1}{2}e^{-2x}$ when

$x \in [0, 1]$. We can check this by substituting this in the differential equation and checking the initial value.

So, $y(1) = \dfrac{1 - e^{-2}}{2} = \dfrac{e^2 - 1}{2e^2} \dots (1)$

Now, for $x \in (1, \infty)$, we have $e^{2x}y = c_2$ (solving the differential equation separately for this interval)

Using the condition found above in (1), we have $c_2 = \dfrac{e^2 - 1}{2}$. That gives $y = \dfrac{e^2 - 1}{2}e^{-2x}$ for $x \in (1, \infty)$

So, for $x = \dfrac{3}{2}$, we get $y = \dfrac{e^2 - 1}{2e^3}$.

So, the correct answer is option A.

#868189

The area (in sq. units) of the region $\{x \epsilon R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$, is

A	$\dfrac{13}{3}$
B	$\dfrac{10}{3}$
C	$\dfrac{5}{3}$
D	$\dfrac{8}{3}$

Solution

The line

$y = x - 2$ intersects

$y = \sqrt{x}$ at

$(4, 2)$.

The area enclosed by the required curve is

$\int_0^4 \sqrt{x}\,dx - \dfrac{1}{2} \times 2 \times 2 = \dfrac{16}{3} - 2 = \dfrac{10}{3}$

So, option B is the correct answer.

#868192

If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta\epsilon\left(0, \dfrac{\pi}{2}\right)$; then $\dfrac{2\cot\beta}{\sin 2\theta}$ is equal to

A	$\sqrt{2}$
B	$\dfrac{2}{\sqrt{3}}$
C	$\dfrac{1}{\sqrt{3}}$
D	$\dfrac{\sqrt{3}}{4}$

Solution

$x^2 + 3y^2 = 9$

$\Rightarrow 2x + 6y\dfrac{dy}{dx} = 0$ .... Differentiating w.r.t x

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-x}{3y}$

Equation of normal is

$-\dfrac{dx}{dy} = \dfrac{3y}{x}$

$\dfrac{dx}{dy}\Big|_{(3\cos\theta,\sqrt{3}\sin\theta)} = \dfrac{3\sqrt{3}\sin\theta}{-3\cos\theta} = \sqrt{3}\tan\theta = m_1$

$\dfrac{dx}{dy}\Big|_{(-3\sin\theta,\sqrt{3}\cos\theta)} = \dfrac{3\sqrt{3}\cos\theta}{-3\sin\theta} = -\sqrt{3}\cot\theta = m_2$

β is the angle between the normals to the ellipse (i), then

$\tan\beta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\dfrac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1 - 3\tan\theta\cot\theta}\right|$

$= \left|\dfrac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1 - 3}\right|$

$\tan\beta = \dfrac{\sqrt{3}}{2}|\tan\theta + \cot\theta|$

$\dfrac{1}{\cot\beta} = \dfrac{\sqrt{3}}{2}|\tan\theta + \cot\theta|$

$\dfrac{1}{\cot\beta} = \dfrac{\sqrt{3}}{2}\left|\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right|$

$\dfrac{1}{\cot\beta} = \dfrac{\sqrt{3}}{2}\left|\dfrac{1}{\sin\theta\cos\theta}\right|$

$$\frac{1}{\cot\beta} = \frac{\sqrt{3}}{\sin 2\theta}$$

$$\Rightarrow \frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

#868194

If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false,m then the truth values of p, q and r are, respectively.

A   F, T, F

B   T, F, T

C   F, F, F

D   T, T, T

Solution

The truth table for the $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false

p	q	r	$\sim p \wedge q$
1	1	1	1
1	0	1	0
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1
0	1	0	1
0	0	0	1

only possible solutions of (p,q,r) is (T,F,T) or (T,F,F)

#868196

The set of all $\alpha \in R$, for which $w = \dfrac{1 + (1 - 8\alpha)z}{1 - z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and $Re\ z \neq 1$, is

A $\{0\}$

B   an empty set

C $\left\{0, \dfrac{1}{4}, -\dfrac{1}{4}\right\}$

D   equal to R

Solution

It is given that $|z| = 1 \& Re\ z \neq 1$

Lets assume $z = x + iy \Rightarrow x^2 + y^2 = 1$ ...(1)

Also,

$$w = \frac{1 + (1 - 8\alpha)z}{1 - z}$$
$$\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy)}{1 - (x + iy)}$$

Multiply and divide above equation by $(1 - x) + iy$, we get

$$w = \frac{(1 + (1 - 8\alpha)(x + iy))((1 - x) + iy)}{(1 - (x + iy))((1 - x) + iy)}$$

Solving it, we get

$$w = \frac{\left[(1 + x(1 - 8\alpha))(1 - x) - (1 - 8x)y^2\right]}{(1 - x)^2 + y^2} + i\frac{[(1 + x(1 - 8\alpha))y - (1 - 8x)y(1 - x)]}{(1 - x)^2 + y^2}$$

It is also given that, w is purely imaginary. Therefore, $Re\ w = 0$

$$\left[(1 + x(1 - 8\alpha))(1 - x) - (1 - 8x)\ ^2\right]$$

$$Re\ w = \frac{\left[(1 + x(1 - 8\alpha))(1 - x) - (1 - 8x)y^2\right]}{(1 - x)^2 + y^2} = 0$$

$$\Rightarrow (1 - x) + x(1 - 8\alpha)(1 - x) = (1 - 8x)y^2$$

$$\Rightarrow (1 - x) + x(1 - 8\alpha) - x^2(1 - 8\alpha) = (1 - 8x)y^2$$

$$\Rightarrow (1 - x) + x(1 - 8\alpha) = 1 - 8\alpha \quad [\because x^2 + y^2 = 1]$$

$$\Rightarrow 1 - 8\alpha = 1$$

$$\Rightarrow \alpha = 0$$

$$\therefore \alpha \in \{0\}$$

#868197

If $\vec{a}, \vec{b},$ and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = 0$, the $|\vec{a} \times \vec{c}|$ is equal to

A $\dfrac{1}{4}$

B $\dfrac{\sqrt{15}}{4}$

C $\dfrac{15}{16}$

D $\dfrac{\sqrt{15}}{16}$

Solution

$$\vec{a} + 2\vec{b} + 2\vec{c} = 0$$

$$\vec{a} + 2\vec{c} = -2\vec{b}$$

$$\Rightarrow (\vec{a} + 2\vec{c}). \vec{a} + 2\vec{c} = -2\vec{b}.(-2\vec{b})$$

$$\vec{a}.\vec{a} + 4\vec{c}.\vec{c} = 4\vec{b}.\vec{b}$$

$$1 + 4 + 4\vec{a}.\vec{c} = 4$$

$$\vec{a}.\vec{c} = \frac{-1}{4}$$

$$= \vec{a}.\vec{c}\hat{c} + \vec{a} \times \vec{c} = \vec{a}$$

$$|\vec{a}.\vec{c}|^2 + |\vec{a} \times \vec{c}|^2 = 1$$

$$\frac{1}{16} + |\vec{a} \times \vec{c}|^2 = 1$$

$$|\vec{a} \times |\vec{c}|^2 = \frac{15}{16}$$

$$|\vec{a} \times |\vec{c}| = \frac{\sqrt{15}}{4}$$

#868199

An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}km$ above it, is observed at an elevation of $60°$ from a point on the ground. If, after five seconds, its elevation from the same point, is $30°$, then the speed (in km/ hr) of the aeroplane, is

A   1500

B   750

C   720

D   1440

Solution

We find the horizontal distance covered by the projection of the plane on the ground in the time given.

We assume that the height at which the plane was flying above ground is constant at $\sqrt{3}$ kms.

In the first case, distance of projection of plane from point of observation is $\dfrac{\sqrt{3}}{\tan 60^o} = 1$ km.

In the second case, the distance of the projection of the plane from the point of observation is $\dfrac{\sqrt{3}}{\tan 30^o} = 3$ km.

So, a distance of $3 - 1 = 2$ km. is covered in 5 seconds. So the speed of the plane is $\dfrac{2 \times 3600}{5} = 1440$ km/hr.

So option D is the correct answer.

#868203

Let
$$S = \left\{ (\lambda, \mu) \epsilon R \times R : f(t) = (|\lambda| e^{|t|} - \mu).\sin(2|t|), t \epsilon R, \text{ is a differentiable function} \right\}.$$
Then S is a subset of?

A $R \times [0, \infty)$

B $(-\infty, 0) \times R$

C $[0, \infty) \times R$

D $R \times (-\infty, 0)$

Solution

$S = \{(\lambda, \mu) \in R \times R: f(t) = (|\lambda| e^{|t|} - \mu) \sin 2|t|, t \in R$

$f(t) = (|\lambda| e^{|t|} - \mu) \sin(2|t|)$

$= \begin{cases} (|\lambda| e^{t} - \mu) \sin et & t > 0 \\ (|\lambda| e^{-t} - \mu)(-\sin 2t) & t < 0 \end{cases}$

$f'(t) = \begin{cases} (|\lambda| e^{t}) \sin 2t + (|\lambda| e^{t} - \mu)(2 \cos et) & t > 0 \\ +|\lambda| e^{-t} \sin 2t + (|\lambda| e^{-t} - \mu)(-\cos 2t) & t < 0 \end{cases}$

Given $f(t)$ is differentiable

$\therefore$ LHD=RHD at $t = 0$

$|\lambda| \cdot \sin 2(0) + (|\lambda| e^{o} - \mu) 2 \cos(\infty)$

$= |\lambda| e^{-0} \sin 2(\infty) - 2 \cos(0)(\lambda| e^{-0} - \mu)$

$0 + (|\lambda| - \mu)2 = 0 - 2(|\lambda| e - \mu)$

$4(|\lambda| - \mu) = 0$

$|\lambda| = \mu$

$S \equiv (\lambda, \mu) = \{\lambda \in R \,\&\, \mu \in (0, \infty)\}$

Set S is subset of $R \times [0, \infty)$

#868204

$n - digit$ numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is

A   6

B   8

C   9

D   7

Solution

Clearly the number of such

n digit numbers is

3^n as each place in the number can be filled by a non-zero digit

$2, 5, 7$.

So we find the least n such that $3^n > 900$. Clearly $n = 7$.

So the correct answer is option D.

#868208

A box $'A'$ contains 2 white, 3 red and 2 black balls. Another box $'B'$ contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box $'B'$ is

A	$\dfrac{7}{16}$
B	$\dfrac{9}{32}$
C	$\dfrac{7}{8}$
D	$\dfrac{9}{16}$

Solution

For the bag

A we can see that there are

2 white,

3 red and

2 black balls. Similarly, from bag

B we have

4 white,

2 red and

3 black balls.

Probability of choosing a white and then a red ball from bag B is given by $= \dfrac{{}^4C_1 \times {}^2C_1}{{}^9C_2}$

Probability of choosing a white ball then a red ball from bag A is given by $= \dfrac{{}^2C_1 \times {}^3C_1}{{}^7C_2}$

So, the probability of getting a white ball and then a red ball from bag B is given by

$$\dfrac{\dfrac{{}^4C_1 \times {}^2C_1}{{}^9C_2}}{\dfrac{{}^4C_1 \times^2 C_1}{{}^9C_2} + \dfrac{{}^2C_1 \times {}^3C_1}{{}^7C_2}} = \dfrac{\dfrac{2}{9}}{\dfrac{2}{7} + \dfrac{2}{9}} = \dfrac{2 \times 7}{18 + 14} = \dfrac{7}{16}$$