#870136

A body of mass $2kg$ slides down with an acceleration of $3m/s^2$ on a rough inclined plane having a slope of 30^o. The external force required to take the same body up the plane with the same acceleration will be: $\left(g = 10m/s^2\right)$

A $4N$

B $14N$

C $6N$

$\boxed{\textbf{D}}$ $20N$

Solution

Equation of motion when the mass slides down

$Mgsin\Theta - f = Ma$

$10 - f = 6$

$f = 4N$

Equation of motion when the block is pushed up

Let the external force be$F$

$F - Mgsin\Theta - f = Ma$

$F - 10 - 4 = 6$

$F = 20N$

#870141

A plane polarized monochromatic EM wave is travelling a vacuum along z direction such that at $t = t_1$ it is found that the electric field is zero at a spatial point z_1 . The next zero that occurs in its neighbouhood is at z_2 . The frequency of the electromagnetic wave is:

$\boxed{\textbf{A}}$ $\dfrac{3 \times 10^8}{|z_2 - z_1|}$

B $\dfrac{6 \times 10^8}{|z_2 - z_1|}$

C $\dfrac{1.5 \times 10^8}{|z_2 - z_1|}$

D $\dfrac{1}{t_1 + \dfrac{|z_2 - z_1|}{3 \times 10^8}}$

Solution

$\in = \in_o - e^{i(kz-wt)}$

at $t = t_1,\ z = z_1,\ E = o,$

the next zero that occurs in it's neighborhood is at z_2 , the frequency of the electromagnetic wave at t_2

$e^{i(kz_1 - wt_1)} = e^{i(kz_2 - wt_2)}$

$kz_1 - wt_1 = kz_2 - wt_2$

$(t_2 - t_1)w = k(z_{-}z_1)$

where $k = \dfrac{2\pi}{\lambda} = 2\pi v$

$$(t_2 - t_1) = \frac{2\pi}{\lambda \times 2\pi v}(z_2 - z_1)$$

$$(t_2 - t_1) = \frac{1}{x \times v}(z_2 - z_1)$$

$$\lambda \times v = \frac{(z_2 - z_1}{(t_1 - t_1}$$

$$C = \frac{(z_2 - z_1)}{(t_2 - t_1)}$$

$$(t_2 - t_1) = \frac{(z_2 - z_1)}{C}$$

Frequency is $f \propto \frac{1}{t}$ then

$$\frac{1}{(t_2 - t_1)} = \frac{C}{(z_2 - z_1)}$$

$$\text{frequency} = \frac{3 \times 10^8}{z_2 - z_1)}$$

#870145

A current of $1A$ is flowing on the sides of an equilateral triangle of side $4.5 \times 10^{-2}m$. The magnetic field at the centre of the triangle will be:

[✓ **A**] $4 \times 10^{-5} Wb/m^2$

B Zero

C $2 \times 10^{-5} Wb/m^2$

D $8 \times 10^{-5} Wb/m^2$

Solution

$l = 4.5 \times 10^{-2}m$

$\tan 60^o = \sqrt{3} = \frac{l}{2d}$

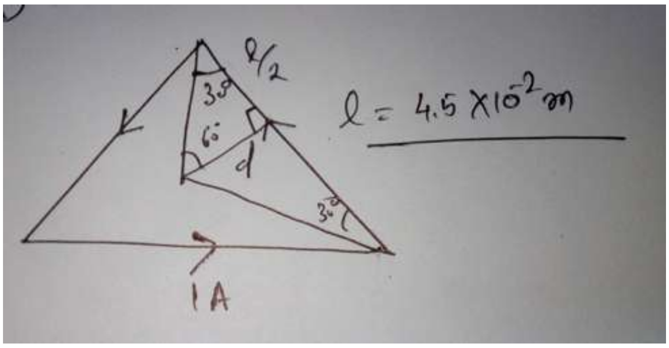
$\Rightarrow d = \frac{l}{2\sqrt{3}} = \left(\frac{4.5 \times 10^{-2}}{2\sqrt{3}}\right) m$

$B = \frac{\mu_o i}{4\pi d}(\cos\theta_1 + \cos\theta_2)$

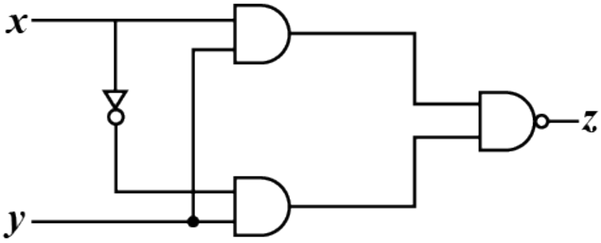
$= \frac{2\mu_0 i}{4\pi d}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\mu_o i}{2\pi}\left(\frac{\sqrt{3}}{2}\right)\frac{2\sqrt{3}}{(4.5 \times 10^{-2})}$

On solving we will get option A as answer

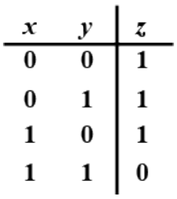


#870163



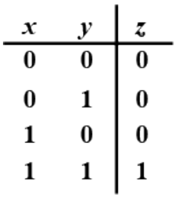
Truth table for the given circuit will be

A



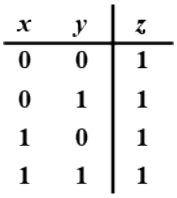
x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

B



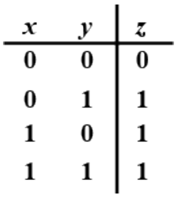
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

C



x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

D



x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

Solution

x	y	$\bar{x}$	$a = x.y$	$b = \bar{x}.y$	$z = \overline{a.b}$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	0	1	0	1

#870165

A constant voltage is applied between two ends of a metallic wire. If the length is halved and the radius of the wire is doubled, the rate of heat developed in the wire will be:

A Increased 8 times

B Doubled

C Halved

D Unchanged

Solution

Rate of heat developed in the wire=

$$P = \frac{V^2}{R}$$

$$R_1 = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

$$P_1 = \frac{V^2}{R_1}$$

$$R_2 = \frac{\rho \frac{L}{2}}{\pi (2r)^2} = \frac{\rho L}{\pi 8 r^2} = \frac{R_1}{8}$$

$$P_2 = \frac{V}{\frac{R_1}{8}} = \frac{8V}{R_1}$$

$$P_2 = 8P_1$$

#870168

The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, h and c. Which of the following correctly gives the Planck length?

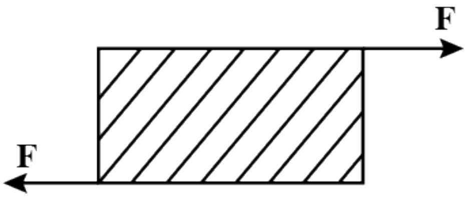
A G^2hc

B $\left(\frac{Gh}{c^3}\right)^{\frac{1}{2}}$

C $G^{\frac{1}{2}} h^2 c$

D Gh^2c^3

#870170



As shown in the figure, forces of $10^5 N$ each are applied in opposite directions, on the upper and lower faces of a cube of side $10 cm$, shifting the upper face parallel to itself by $0.5 cm$. If the side of another cube of the same material is $20 cm$, then under similar conditions as above, the displacement will be:

A $1.00 cm$

[✓ **B**] $0.25 cm$

C $0.37 cm$

D $0.75 cm$

Solution

For same material the ration of stress to strain is same

For first cube

$$Stress_1 = \frac{10^5}{(0.1^2)}$$
$$strain_1 = \frac{0.5 \times 10^{-2}}{0.1}$$

For second block,

$$stress_2 = \frac{10^5}{(0.2)^2}$$
$$strain_2 = \frac{x}{0.2}$$

where x is the displacement for second block.

For same material,

$$\frac{stress_1}{strain_1} = \frac{stress_2}{strain_2}$$

From this $x = 0.25 cm$

#870173

The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance $49 \mu H$ and a capacitance of $2.5 nF$. It is modulated by an audio signal of $12 kHz$. The frequency range occupied by the side bands is:

A $18 kHz - 30 kHz$

B $63 kHz - 75 kHz$

[✓ **C**] $442 kHz - 466 kHz$

D $13482 kHz - 13494 kHz$

Solution

$$w = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{49 \times 10^{-6} \times \frac{2.5}{10} \times 10^{-9}}}$$

$$= \frac{1}{7 \times 5 \times 10^{-8}} = \frac{10^8}{7 \times 5} = w$$

$$= \frac{10^8}{7 \times 5} = 2\pi \times f = 2 \times \frac{22}{7} \times f$$

$$\frac{10^8}{22 \times 10} = f$$

$$\frac{10^7}{22} = f$$

$$\frac{10^4}{22} kHz = f$$

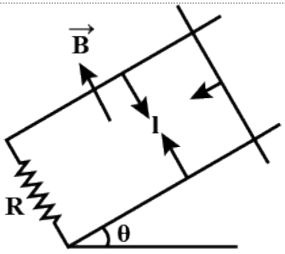
$f = 454.54 kHz$

for frequency range

$454.54 \pm 12 kHz$

$442 kHz - 466 kHz$

#870177



A copper rod of mass m slides under gravity on two smooth parallel rails, with separation 1 and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R.There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is:

A $\frac{mgR\cos\theta}{B^2l^2}$

$\boxed{\textbf{B}}$ $\frac{mgR\sin\theta}{B^2l^2}$

C $\frac{mgR\tan\theta}{B^2l^2}$

D $\frac{mgR\cot\theta}{B^2l^2}$

Solution

$$\in = \frac{d\phi}{dt} = \frac{d(BA)}{lt}$$

$$= \frac{d(Bl)}{dt}$$

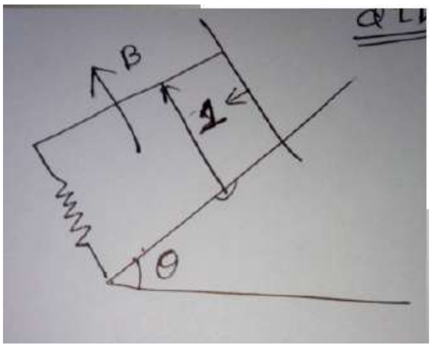
$$= \frac{Bdl}{dt} = BVl$$

$$F = ilB = \left(\frac{BV}{R}\right)(l^2B) = \frac{B^2l^2V}{R}$$

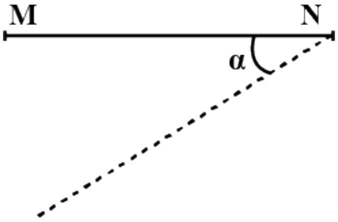
At equilibrium

$$\Rightarrow mg\sin\theta = \frac{B^2lV}{R}$$

$$\Rightarrow V = \frac{mg\,R\sin\theta}{B^2l^2}$$



#870178



A think rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to: (see figure)

[✓ **A**] $\sqrt{\cos\alpha}$

B $\cos\alpha$

C $\sin\alpha$

D $\sqrt{\sin\alpha}$

Solution

When the rod makes an angle of

α displacement of centre of mass

$= \frac{l}{2} cos\alpha$

$mg\frac{l}{2}cos\alpha = \frac{1}{2}I\omega^2$

$mg\frac{l}{2}cos\alpha = \frac{ml^2}{6}\omega^2$

$\omega = \sqrt{\frac{3gcos\alpha}{l}}$

speed of end= $\omega \times l$=$\sqrt{3gcos\alpha l}$

hence ω is proportional to $\sqrt{cos\alpha}$

#870179

A parallel plate capacitor with area $200cm^2$ and separation between the plates $1.5cm$, is connected across a battery of emf V. If the force of attraction between the plates is $25 \times 10^{-6}N$, the value of V is approximately:

$\left(\epsilon_0 = 8.85 \times 10^{--12}\frac{C^2}{N.m^2}\right)$

A $150V$

B $100V$

[✓ **C**] $250V$

D $300V$

Solution

$A = 200cm^2$

$d = 1.5cm$

$F = 25 \times 10^{-6}N$

$\because E = \frac{\sigma}{2 \in_o} = \frac{Q}{2A \in_o}$

$F = QE$

$F = \frac{Q^2}{2A \in_o}$

But $Q = CV = \frac{\in_o A(V)}{d}$

$\therefore F = \frac{(\in_o AV)^2}{d^2 \times 2A \in_o}$

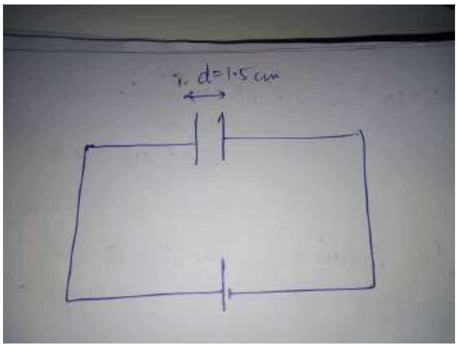
$$= \frac{(\in_o A)^2 \times V^2}{d^2 \times 2 \times (A \in_o)}$$

$$= \frac{(\in_o A)V^2}{d^2 \times 2}$$

$$25 \times 10^{-6} = \frac{(8.85 \times 10^{-12}) \times (200 \times 10^{-4}) \times V^2}{2.25 \times 10^{-4} \times 2}$$

$$V = \sqrt{\frac{25 \times 10^{-6} \times 2.25 \times 10^{-4} \times 2}{8.85 \times 10^{-12} \times 200 \times 10^{-4}}}$$

Here, on solving, $v \approx 250V$



#870181

A solid ball of radius R has a charge density ρ given by $\rho = \rho_0 \left(1 - \frac{r}{R}\right)$ for $0 \leq r \leq R$. The electric field outside the ball is:

A $\frac{\rho_0 R^3}{\epsilon_0 r^2}$

B $\frac{4\rho_0 R^3}{3\epsilon_0 r^2}$

C $\frac{3\rho_0 R^3}{4\epsilon_0 r^2}$

D $\frac{\rho_0 R^3}{12\epsilon_0 r^2}$

Solution

$\rho = \rho_0 \left(1 - \frac{r}{R}\right)$

$dq = \rho dv$

$q_{in} = \int dq$

$= \rho dv$

$\rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$

$\because dv = 4\pi r^2 dr$

$$= 4\pi\rho_0 \int_0^R \left(1 - \frac{r}{R}\right) r^2 dr$$

$$= 4\pi\rho_o \int_o^R r^2 dr - \frac{r^2}{R} dr$$

$$= 4\pi\rho_o\left[\left[\frac{r^3}{3}\right]_o^R - \left[\frac{r^4}{4R}\right]_o^R\right]$$

$$= 4\pi\rho_o\left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$= 4\pi\rho_o\left[\frac{R^3}{3} - \frac{R^3}{4}\right]$$

$$= 4\pi\rho_o\left[\frac{R^3}{12}\right]$$

$$q = \frac{\pi\rho_o R^3}{3}$$

$$E.4\pi r^2 = \left(\frac{\pi\rho_o R^3}{3 \in_o}\right)$$

$$\Rightarrow E = \frac{\rho_o R^3}{12 \in_o r^2}$$

#870184

A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90^o with respect to each other. The mass of unknown particle is:

A $\frac{m}{\sqrt{3}}$

B $\frac{m}{2}$

C $2m$

$\boxed{\text{D}}$ m

Solution

Apply principle of conservation of momentum along x-direction,

$mu = mv_1 \cos 45 + Mv_2 \cos 45$

$mu = \frac{1}{\sqrt{2}}(mv_1 + Mv_2)$ ...(1)

along $y-$ direction,

$o = mv_1 \sin 45 - Mv_2 \sin 45$

$o = (mv_1 - Mv_2)\frac{1}{\sqrt{2}}$ ...(2)

Coefficient of $e = 1 = \frac{v_2 - v_1 \cos 90}{u \cos 45}$

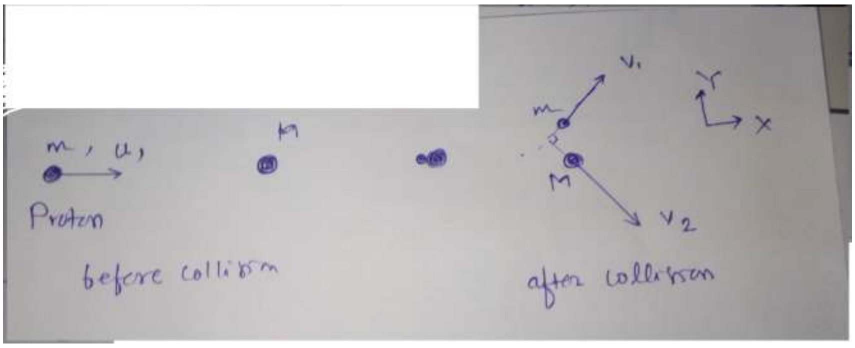
Restution

$\Rightarrow \frac{v_2}{\frac{u}{\sqrt{2}}} = 1$

$\Rightarrow u = \sqrt{2}v_2$ ...(3)

solving eqn (1), (2), & (3) we get

$M = m$



#870186

A disc rotates about its axis of symmetry in a hoizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of $1.25cm$ from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is $\left(g = 10m/s^2\right)$

A 0.5

B 0.7

C 0.3

D 0.6

Solution

$3.5\ rev/second$

$1rev \rightarrow 2\pi\ rad$

$3.5\ rev \rightarrow 2\pi \times 3.5\ rad$

$\Rightarrow w = 7\pi\ rad/sec$

$\mu mg = \dfrac{mv^2}{1.25}$

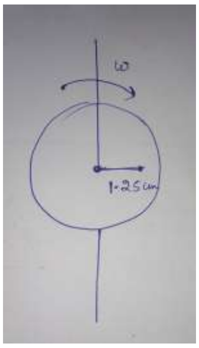
$\mu mg = \dfrac{m(rw)^2}{r}$

$\mu mg = mrw^2$

$$\mu = \frac{rw^2}{g} = \frac{1.25 \times 10^{-2} \times \left(7 \times \frac{22}{7}\right)^2}{10}$$

$$= \frac{1.25 \times 10^{-2} \times 22^2}{10}$$

$$= 0.6$$



#870190

At the centre of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time t of its start of rotation.

A $\dfrac{\mu_0 I}{2R}\omega r^2 \sin \omega t$

B $\dfrac{\mu_0 I}{4R}\omega \pi r^2 \sin \omega t$

C $\dfrac{\mu_0 I}{2R}\omega \pi r^2 \sin \omega t$

D $\dfrac{\mu_0 I}{4R}\omega r^2 \sin \omega t$

Solution

$\phi = \overline{B}.\overline{A} = BA \cos wt = \pi r^2 b \cos wt$

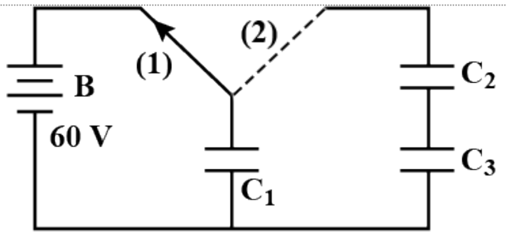
$\in = -\dfrac{d\phi}{dt}$

$= -\dfrac{d}{dt}(\pi r^2 B \cos wt)$

$= \pi r^2 B \sin wt(w)$

$= \dfrac{\mu_o I}{2R}\pi w r^2 \sin wt \ \left(\therefore B = \dfrac{\mu_o I}{2R}\right)$

#870227



A capacitor $C_1 = 10\mu F$ is charged up to a voltage $V = 60V$ by connecting it to battery B through switch (1), Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors $C_2 = 3.0\mu F$ and $PC_3 = 6.0\mu F$ through a switch (2) as shown in the figure. The sum of final charges on C_2 and C_3 is:

[✓ **A**] $36\mu C$

B $20\mu C$

C $54\mu C$

D $40\mu C$

#870231

5 beats/ second are heard when a turning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either $0.95m$ or $1m$. The frequency of the fork will be:

[✓ **A**] $195Hz$

B $251Hz$

C $150Hz$

D $300Hz$

Solution

$L_1 = 0.95m, L_2 = 1m$

$L_2 > L_1, n_1 > N > n_2$

$n_1 - N = 5andN - n_2 = 5$

on solving $n_1 - n_2 = 10$

$n_2 = n_1 - 10$

By law of length of vibrating string

$n_1L_1 = n_2L_2$

On solving we get $n_1 = 200Hz$

$n_1 - N = 5$

N = 195 Hz

#870235

Two simple harmonic motions, as shown, are at right angles. They are combined to form Lissajous figures.

$x(t) = A\sin(at + \delta)$

$y(t) = B\sin(bt)$

Identify the correct match below

A Parameters: $A = B, a = 2b; \delta = \frac{\pi}{2}$; Curve: Circle

B Parameters: $A = B, a = b; \delta = \frac{\pi}{2}$; Curve: Line

[✓ **C**] Parameters: $A \neq B, a = b; \delta = \frac{\pi}{2}$; Curve: Ellipse

D Parameters: $A \neq B, a = b; \delta = 0$; Curve: Parabola

Solution

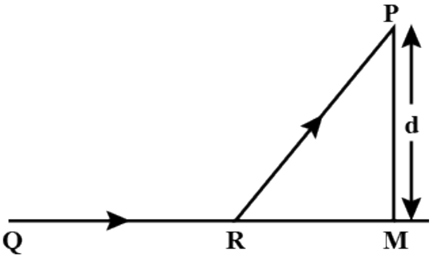
Lissajous curves take common shapes depending on the variables in the expressions.

$x = A\sin(at + \delta)$

$y = B\sin(bt + r)$

If $A \neq B$ & $a = b$ we obtain ellipse

#870239



A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?

A $\frac{d}{\sqrt{3}}$

[✓ **B**] $\frac{d}{2}$

C $\frac{d}{\sqrt{2}}$

D d

Solution

Let the car turn of the highway at a distance 'x' from the point

M. So,

$RM = x$

And if speed of car in field is v, then time taken by the car to cover the distance $QR = QM - x$ on the highway, $t_1 = \frac{QM - x}{2v}$ ....(1)

the time taken to travel the distance 'RP' in the field

$t_2 = \frac{\sqrt{d^2 + x^2}}{v}$ ...(2)

So, the total time elapsed to move the car from Q to P

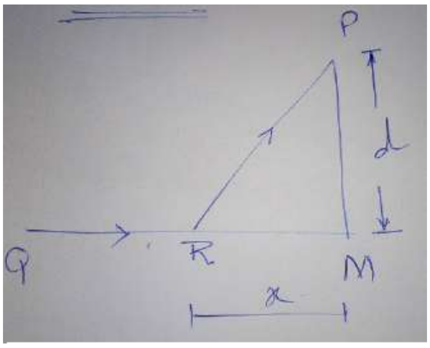
$$t = t_1 + t_2 = \frac{QM - x}{2v} + \frac{\sqrt{d^2 + x^2}}{v}$$

for 't' to be minimum

$$\frac{dt}{dx} = 0$$

$$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$$

or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$



#870251

A copper rod of cross-sectional area A carries a uniform current I through if. At temperature T, if the volume charge density of the rod is ρ, how long will the charges take to travel a distance d?

A $\frac{2\rho dA}{IT}$

B $\frac{2\rho dA}{I}$

$\boxed{\textbf{C}}$ $\frac{\rho dA}{I}$

D $\frac{\rho dA}{IT}$

Solution

$$\rho = \frac{q}{vol}$$
$$= \frac{q}{Ad}$$
$$q = \rho Ad$$
$$q = it$$
$$t = \frac{q}{I}$$
$$= \frac{\rho Ad}{I}$$

#870260

Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at $600K$ and rejects heat to a reservoir heat to reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at $100K$. If the efficiencies of the two engines A and B are represented by η_A and η_B respectively, then what is the value of $\frac{\eta_A}{\eta_B}$

$\boxed{\textbf{A}}$ $\frac{12}{7}$

B $\frac{12}{5}$

C $\frac{5}{12}$

D $\frac{7}{12}$

#870261

A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal length of $10cm$. The separation between the two lenses is $2cm$. The focal lengths of the component lenses

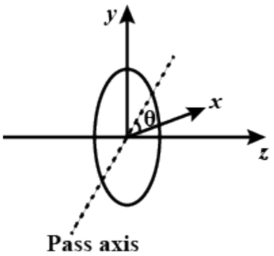
$\boxed{\textbf{A}}$ $18cm, 20cm$

B $10cm, 12cm$

C $12cm, 14cm$

D $16cm, 18cm$

#870265



A plane polarized light is incident on a polariser with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of $\theta, \theta = 8^o, 38^o, 188^o$ and 218^o, the observed intensities are same. What is the angle between the direction of polarization and x-axis

[✓ A] 203^o

B 45^o

C 98^o

D 128^o

#870266

An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of $8 : 27$. The ratio of the radii of the nuclei (assumed to be spherical) is:

A $8 : 27$

B $2 : 3$

[✓ C] $3 : 2$

D $4 : 9$

Solution

$$\frac{V_1}{V_2} = \frac{8}{27}$$

$$m_1 V_1 = m_2 V_2$$

$$\frac{m_1}{m_2} = \frac{V_2}{V_1} = \frac{27}{8}$$

$$\frac{\rho \times \frac{4}{3}\pi R_1^3}{\rho \times \frac{4}{3}\pi R_2^3}$$

$$\left(\frac{R_1}{R_2}\right) = \left(\frac{27}{8}\right)^{\frac{1}{3}}$$

$$= \left(\frac{3}{2}\right)^{3\times\frac{1}{3}}$$

$$\frac{R_1}{R_2} = \frac{3}{2}$$

#870269

A body takes 10 minutes to cool from 60^oC to 50^oC. The temperature of surroundings is constant at 25^oC. Then, the temperature of the body after next 10 minutes will be approximately

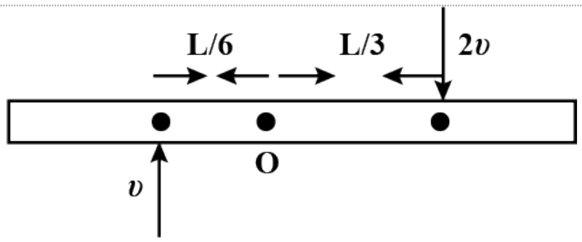
[✓ A] 43^oC

B 47^oC

C 41^oC

D 45^oC

#870272



A thin uniform bar of length L and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane from opposite sides of the bar with speeds $2v$ and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be:

[✓ **A**] $\frac{v}{6L}$

B $\frac{6v}{5L}$

C $\frac{3v}{5L}$

D $\frac{v}{5L}$

#870276

If the de Broglie wavelengths associated with a proton and an $\alpha-$ particle are equal, then the ratio of velocities of the proton and the $\alpha-$ particle will be:

A $1:4$

B $1:2$

[✓ **C**] $4:1$

D $2:1$

Solution

Given de Broglie Wavelength

$\lambda_p = \lambda_\alpha$

So, $\frac{h}{m_p \times v_p} = \frac{h}{m_\alpha \times v_\alpha}$

$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p}$

Because Mass of α particle is 4 times mass of proton

So 4:1

Option C is correct answer

#870282

When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to $10m$ height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature):

A $10.5m$

B $8.7m$

C $11.2m$

[✓ **D**] $9.5m$

Solution

Pressure at bottom

$(P_1) = P_{atm} + \rho gh + \frac{4T}{R_1}$ ____________(1)

Pressure at top

$(P_2) = P_{atm} + \frac{4T}{R_2}$ ________________(2)

Given $R_1 = r$

$R_2 = \frac{5r}{4}$

So $P_1V_1 = P_2V_2$

$(P_1)\frac{4}{3}\pi r^3 = (P_2)\frac{4}{3}\frac{125r^3}{64}$

Dividing (1) and (2)

$$\frac{P_1}{P_2} = \frac{P_{atm} + \rho gh + \frac{4T}{r}}{P_{atm} + \frac{4T \times 4}{5r}} = \frac{125}{64}$$

$$\frac{\rho g(10) + \rho gh}{\rho g(10)} = \frac{125}{64}$$

$$640 + 64h = 1250$$

On solving we get h = 9.5 m

#870298

Muon $\left(\mu^{-1}\right)$ is negatively charged ($|q| = |e|$) with a mass $m_\mu = 200m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^{-1} is bound to a proton to form a hydrogen like atom, identify the correct statements

(A) Radius of the muonic orbit is 200 times smaller than that of the electron

(B) the speed of the μ^{-1} in the nth orbit is $\frac{1}{200}$ times that of the electron in the nth orbit

(C) The ionization energy of muonic atom is 200 times more than that of an hydrogen atom

(D) The momentum of the muon in the nth orbit is 200 times more than that of the electron

$\boxed{\text{A}}$ (A), (B), (D)

B (B), (D)

C (C), (D)

D (A), (C), (D)

Solution

(A) Radius of muon

$$= \frac{\text{Radius of hydrogen}}{200}$$

Radius of H atom $= r = \dfrac{\in_o n^2 h^2}{\pi m e^2}$

Radius of muon $= r_\mu = \dfrac{\in_o n^2 h^2}{\pi \times 200 m e^2}$

$$r_\mu = \frac{r}{200}$$

(B) Velocity relation given is wrong

(C) Ionization energy in e^- H atom

$$E = \frac{+me^4}{8 \in -o^2 n^2 h^2}$$

$$E_\mu = \frac{200me^4}{8 \in_o^2 n^2 h^2} = 200E$$

(D) Momentum of H-atom

$$mvr = \frac{nh}{2\pi}$$

momentum of muon is $200 \times mvr$

hence (A), (C) & (D) are correct

#870300

The value closest to the thermal velocity of a Helium atom at room temperature ($300K$) in ms^{-1} is:

$\left[k_B = 1.4 \times 10^{-23} J/K; m_{He} = 7 \times 10^{-27} kg\right]$

$\boxed{\text{A}}$ 1.3×10^4

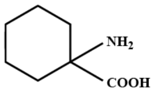
B 1.3×10^5

C 1.3×10^2

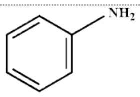
D 1.3×10^3

#870099

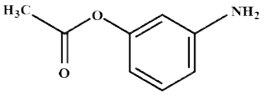
(A)



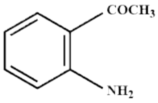
(B)



(C)



(D)



The increasing order of diazotisation of the following compounds is?

A (d) $<$ (c) $<$ (b) $<$ (a)

[✓ **B**] (a) $<$ (d) $<$ (b) $<$ (c)

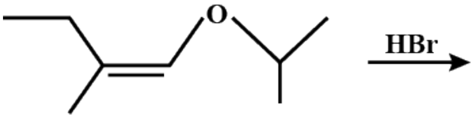
C (a) $<$ (b) $<$ (c) $<$ (d)

D (a) $<$ (d) $<$ (c) $<$ (b)

Solution

Aromatic diazonium salts are more stable than aliphatic diazonium salts. The stability of aryl diazonium salts is due to resonance. Electron donating substituents increase electron density on benzene ring. They increase the stability of diazonium salts. Electron withdrawing substituents decrease electron density on benzene ring. They decrease the stability of diazonium salts. $-COCH_3$ group is electron withdrawing and hence, (d) is less stable than (b). Although $-O-COCH_3$ is electron donating substituent, but it is present in meta position. Hence, it will not have significant effect on stability. The increasing order of diazotisation is $(a)<(d)<(b)<(c)$.

#870101



The total number of optically active compounds formed in the following reaction is?

A Zero

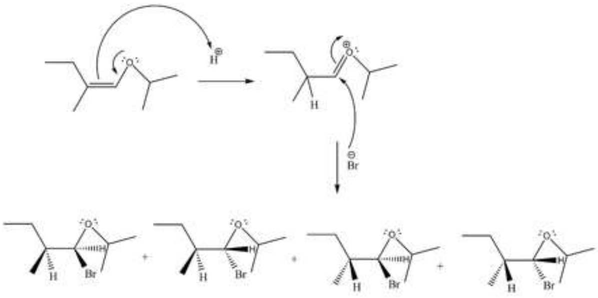
B Six

[✓ **C**] Four

D Two

Solution

The total number of optically active compounds formed is four. The product has two chiral C atoms (marked with asterik). Thus, it has $2^n=2^2=4$ stereoisomers.



#870104

In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are, respectively.

A Superoxide and -1

[✓ B] Superoxide and $-1/2$

C Peroxide and $-1/2$

D Oxide and -2

Solution

In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are,superoxide and $-1/2$ respectively.

Superoxide ion is O_2^-.

Let X be oxidation state of oxygen. The oxidation state of K is +1.

$+1+2(X)=0$

$2X=-1$

$X=-\frac{1}{2}$

#870110

$\Delta_f G^o$ at 500 K for substance 'S' in liquid state and gaseous state are $+100.7$ kcal mol^{-1} and $+103$ kcal mol^{-1}, respectively. Vapour pressure of liquid 'S' at 500 K is approximately equal to: ($R=2$ cal $K^{-1}mol^{-1}$).

A 100 atm

B 1 atm

[✓ C] 10 atm

D 0.1 atm

Solution

$\Delta G^o_{rxn}=\Delta_f G^o(\text{vapour})-\Delta_f G^o(\text{liquid})$

$\Delta G^o_{rxn}=103-100.7=2.3$ kcal/mol

$\Delta G^o_{rxn}=-RTlnK$

$2.3kcal/mol\times 1000cal/kcal=-2cal/mol/K\times 500K\times lnK$

$lnK=2.3$

$K=10atm=$ Vapour pressure of liquid 'S'

Vapour pressure of liquid 'S' at 500 K is approximately equal to 10 atm.

#870113

In XeO_3F_2, the number of bond pair(s), π-bond(s) and lone pair(s) on Xe atom respectively are.

[✓ A] $5,3,0$

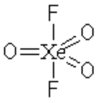
B $5,2,0$

C $4,2,2$

D $4,4,0$

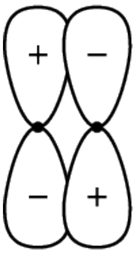
Solution

In XeO_3F_2, the number of bond pair(s), π-bond(s) and lone pair(s) on Xe atom are $5,3,0$ respectively. There are three $Xe=O$ double bonds and two $Xe-F$ single bonds. All the valence electrons of Xe are involved in bonding.



XeO_3F_2

#870117



Which of the following best describes the diagram of a molecular orbital?

A A bonding π orbital

B A non-bonding orbital

C An antibonding σ orbital

[✓ **D**] An antibonding π orbital

Solution

An antibonding π orbital best describes the diagram of a molecular orbital. Two p orbitals laterally overlap to form pi bond. Out of phase combination of these two p orbitals give π^* MO.

#870120

Following four solutions are prepared by mixing different volumes of NaOH and HCl of different concentrations, pH of which one of them will be equal to 1?

A 55mL $\frac{M}{10}$ HCl $+45$mL $\frac{M}{10}$ NaOH

[✓ **B**] 75mL $\frac{M}{5}$ HCl+25mL $\frac{M}{5}$ NaOH

C 100mL $\frac{M}{10}$ HCl+100mL $\frac{M}{10}$ NaOH

D 60mL $\frac{M}{10}$ HCl+40mL $\frac{M}{10}$ NaOH

Solution

75mL $\frac{M}{5}$ HCl+25mL $\frac{M}{5}$ NaOH
25mL $\frac{M}{5}$ NaOH will neutralise 25 mL $\frac{M}{5}$ HCl
$75-25=50$ mL $\frac{M}{5}$ HCl will remain.
Total volume will be $75+25=100$ mL.
50 mL $\frac{M}{5}$ HCl is diluted to 100 mL.
$[H^+]=[HCl]=\frac{M}{5}\times\frac{50}{100}=\frac{M}{10}$
$pH=-log_{10}[H^+]=-log_{10}\frac{M}{10}=1$

#870123

Given
(i) $2Fe_2O_3(s)\rightarrow 4Fe(s)+3O_2(g)$;
$\Delta_rG^o=+1487.0$ kJ mol^{-1}
(ii) $2CO(g)+O_2(g)\rightarrow 2CO_2(g)$;
$\Delta_rG^o=-514.4$kJ mol^{-1}
Free energy change, Δ_rG^o for the reaction
$2Fe_2O_3(s)+6CO(g)\rightarrow 4Fe(s)+6CO_2(g)$ will be.

A -112.4kJ mol^{-1}

[✓ **B**] -56.2 kJ mol^{-1}

C -208.0 kJ mol^{-1}

D -168.2 kJ mol^{-1}

Solution

(i) $2Fe_2O_3(s)\rightarrow 4Fe(s)+3O_2(g)$;

$\Delta_r G^o = +1487.0$ kJ mol^{-1}

(ii) $2CO(g) + O_2(g) \rightarrow 2CO_2(g);$

$\Delta_r G^o = -514.4$kJ mol^{-1}

Multiply above reaction with 3

(iii) $6CO(g) + 3O_2(g) \rightarrow 6CO_2(g);$

$\Delta_r G^o = 3 \times -514.4 = -1543.2$kJ mol^{-1}

When we add reaction (i) and reaction (iii), we get reaction (iv)

(iv)$2Fe_2O_3(s) + 6CO(g) \rightarrow 4Fe(s) + 6CO_2(g)$

Free energy change, $\Delta_r G^o$ for the reaction will be.

$1487.0 - 1543.2 = -56.2$ kJ mol^{-1}

#870125

At a certain temperature in a 5L vessel, 2 moles of carbon monoxide and 3 moles of chlorine were allowed to reach equilibrium according to the reaction, $CO + Cl_2 \rightleftharpoons COCl_2$

At equilibrium, if one mole of CO is present then equilibrium constant (K_c) for the reaction is?

A 2.5

B 4

C 2

D 3

Solution

Initially, 2 moles of CO are present.

At equilibrium, 1 mole of CO is present.

Hence, $2 - 1 = 1$ mole of CO has reacted.

1 mole of CO will react with 1 mole of Cl_2 to form 1 mole of $COCl_2$.

$3 - 1 = 2$ moles of Cl_2 remains at equilibrium.

The equilibrium constant

$$K_c = \frac{[COCl_2]}{[CO][Cl_2]}$$

$$K_c = \frac{\frac{1 \text{ mol}}{5 \text{ L}}}{\frac{1 \text{ mol}}{5 \text{ L}} \times \frac{2 \text{ mol}}{5 \text{ L}}}$$

$$K_c = 2.5$$

#870129

The correct order of spin-only magnetic moments among the following is?(Atomic number: $Mn = 25, Co = 27, Ni = 28, Zn = 30$).

A $[ZnCl_4]^{2-} > [NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$

B $[CoCl_4]^{2-} > [MnCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

C $[NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-} > [ZnCl_4]^{2-}$

D $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

Solution

The complex having higher number of unpaired electrons will have higher value of spin-only magnetic moment.

The correct order of spin-only magnetic moments is $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$.

In theses complexes, the central metal ion is in $+2$ oxidation state.

Zn^{2+} has $3d^{10}$ outer electronic configuration with 0 unpaired electrons.

Ni^{2+} has $3d^8$ outer electronic configuration with 2 unpaired electrons.

Co^{2+} has $3d^7$ outer electronic configuration with 3 unpaired electrons.

Mn^{2+} has $3d^5$ outer electronic configuration with 5 unpaired electrons.

#870131

When 2-butyne is treated with H_2/Lindlar's catalyst, compound X is produced as the major product and when treated with Na/liq. NH_3 it produces Y as the major product. Which of the following statements is correct?

A Y will have higher dipole moment and higher boiling point than X

B Y will have higher dipole moment and lower boiling point than X

C X will have lower dipole moment and lower boiling point than Y

[✓ **D**] X will have higher dipole moment and higher boiling point than Y

Solution

When 2-butyne is treated with H_2 /Lindlar's catalyst, compound X (cis-2-butene) is produced as the major product and when treated with Na/liq. NH_3 it produces Y (trans-2-butene) as the major product. X will have higher dipole moment and higher boiling point than Y. Cis isomer will have higher dipole moment and higher boiling point than trans. trans-2-butene has center of inversion and hence, zero dipole moment.

The boiling points of cis and trans isomers of 2-butene are 277 K and 274 K respectively.

#870134

For a first order reaction, $A \rightarrow P, t_{1/2}$ (half-life) is 10 days. The time required for $\frac{1^{th}}{4}$ conversion of A(in days) is: (ln $2 = 0.693$, ln $3 = 1.1$).

A 3.2

B 2.5

[✓ **C**] 4.1

D 5

Solution

The half life $t_{1/2} = 10$ days

The decay constant $k = \frac{0.693}{t_{1/2}} = \frac{0.693}{10 days} = 0.0693 day^{-1}$

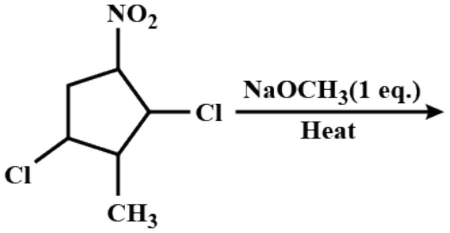
The time required for one fourth conversion

$t = \frac{2.303}{k} log_{10} \frac{a}{a-x}$

$t = \frac{2.303}{0.0693 day^{-1}} log_{10} \frac{1}{1-(1/4)}$

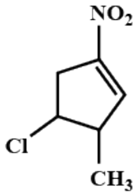
$t = 4.1$ days

#870137

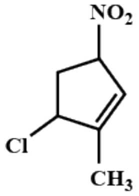


The major product formed in the following reaction is?

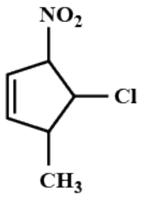
[✓ **A**]



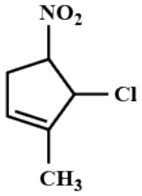
B



C



D



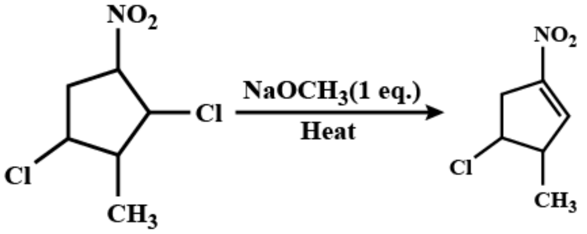
Solution

The major product formed is represented by option (A).

Nitro group is electron withdrawing group. Hence, increases the acidity of H atom (attached to C atom bearing nitro group).

Hence, removal of H becomes easy. Also, the newly formed double bond is in conjugation with nitro group.

Note: In the given reaction, a molecule of HCl is lost and $C=C$ double bond is formed. THus, it is dehydrohalogenation reaction.



#870140

The de-Broglie's wavelength of electron present in first Bohr orbit of 'H' atom is?

A $4\times 0.529\overset{o}{A}$

[✓ B] $2\pi\times 0.529\overset{o}{A}$

C $\frac{0.529}{2\pi}\overset{o}{A}$

D $0.529\overset{o}{A}$

Solution

The de-Broglie's wavelength of electron present in first Bohr orbit of 'H' atom is $2\pi\times 0.529\overset{o}{A}$.

First Bohr orbit of 'H' atom has radius $r=0.529\ A^o$

Also, the angular momentum is quantised.

$$mvr=\frac{h}{2\pi}$$
$$2\pi r=\frac{h}{mv}=\lambda$$
$$\lambda=2\pi\times 0.529\overset{o}{A}$$

#870143

Lithium aluminium hydride reacts with silicon tetrachloride to form.

A LiCl, AlH_3 and SiH_4

[✓ B] LiCl, $AlCl_3$ and SiH_4

C LiH, $AlCl_3$ and $SiCl_2$

D LiH, AlH_3 and SiH_4

Solution

Lithium aluminium hydride reacts with silicon tetrachloride to form LiCl (lithium chloride),

$AlCl_3$ (aluminum trichloride) and SiH_4 (silicon hydride).

$$SiCl_4+LiAlH_4\rightarrow LiCl+AlCl_3+SiH_4$$

#870147

The correct order of electron affinity is?

A O > F > Cl

B F > O > Cl

C F > Cl > O

$\boxed{\textbf{D}}$ Cl > F > O

Solution

The correct order of electron affinity is $Cl > F > O$

On moving from left to right across a period, the electron affinity becomes more negative.

On moving from top to bottom in a group, the electron affinity becomes less negative.

Chlorine has more negative electron affinity than fluorine. Because adding an electron to fluorine 2p orbital causes greater repulsion than adding an electron to chlorine 3p orbital which is larger in size.

Note: The electron affinity (in kJ/mol) of Cl, F and O is -349, -328 and -141 respectively.

#870153

Two 5 molal solutions are prepared by dissolving a non-electrolyte non-volatile solute separately in the solvents X and Y. The molecular weights of the solvents are M_X and M_Y, respectively where $M_X = \frac{3}{4}M_Y$. The relative lowering of vapour pressure of the solution in X is "m" times that of the solution in Y. Given that the number of moles of solute is very small in comparison to that of solvent, the value of "m" is?

$\boxed{\textbf{A}}$ $\frac{3}{4}$

B $\frac{1}{2}$

C $\frac{1}{4}$

D $\frac{4}{3}$

Solution

THe relationship between molar masses of two solvents is

$M_X = \frac{3}{4}M_Y$ ......(1)

THe relative lowering of vapour pressure of two solutions is

$(\frac{\Delta P}{P})_X = m(\frac{\Delta P}{P})_Y$

But, the relative lowering of vapour pressure of solution is directly proportional to the mole fraction of solute.

$M_x \times \frac{5}{1000} = m \times M_Y \times \frac{5}{1000}$ .....(2)

Substitute equation (1) in equation (2).

$\frac{3}{4} \times M_Y \times \frac{5}{1000} = m \times M_Y \times \frac{5}{1000}$

$m = \frac{3}{4}$

Note:

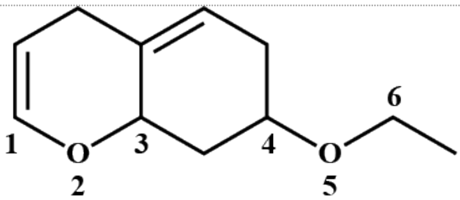
5 molal solution means 5 moles of solute are dissolved in 1 kg (or 1000 g) of solvent. The number of moles of solvent $= \frac{1000g}{M}$

The mole fraction of solute

$= \frac{5}{1000/M}$

$= M \times \frac{5}{1000}$

#870154



On the treatment of the following compound with a strong acid, the most susceptible site for bond cleavage is?

A $O2 - C3$

[✓ **B**] $O5 - C6$

C $C4 - O5$

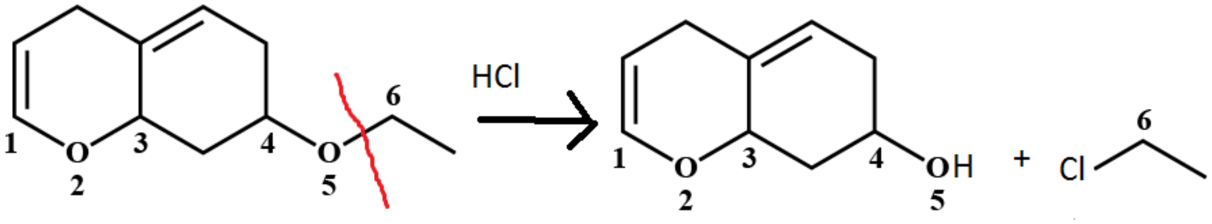
D $C1 - O2$

Solution

On the treatment of the given compound with a strong acid, the most susceptible site for bond cleavage is $O5 - C6$.

The lone pair of electrons on $O2$ is involved in resonance with $C = C$. Hence, $O2$ will not be protonated.

The lone pair of electrons on $O5$ is not involved in resonance with $C = C$. Hence, $O5$ will be protonated. Chloride ion will then attack least substituted C atom $(C6)$



#870156

All of the following share the same crystal structure except.

A RbCl

B NaCl

C CsCl

[✓ **D**] LiCl

Solution

RbCl, NaCl and CsCl share the same crystal structure except LiCl.

LiCl is deliquescent. It crystalises as a hydrate $LiCl \cdot 2H_2O$. Other alkali metal chlorides do not form hydrates.

#870158

The total number of possible isomers for square-planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is?

A 16

[✓ **B**] 12

C 8

D 24

Solution

The total number of possible isomers for square-planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is 12.

THe square-planar complex of type

$[Mabcd]^{n\pm}$, where all four ligands are different, has 3 geometrical isomers. But if one of the ligands is ambidentate, then $2 \times 3 = 6$ geometrical isomers are possible. But if two ligands are ambidentate, then $4 \times 3 = 12$ geometrical isomers are possible.

In our example, both NO_2^- and SCN^- are ambidentate ligands.

#870160

Two compounds I and II are eluted by column chromatography(adsorption of I > II). Which one of the following is a correct statement?

A II moves slower and has higher R_f value than I

[✓ **B**] II moves faster and has higher R_f value than I

C I moves faster and has higher R_f value than II

D I moves slower and has higher R_f value than II

Solution

Two compounds I and II are eluted by column chromatography(adsorption of I $>$ II). The statement (B) is a correct statement.

II moves faster and has higher R_f value than I

Since, adsorption of I $>$ II, I is firmly attached to column (stationary phase). Hence, it will move slowly and will move little distance. Also, II is loosely attached to column (stationary phase). Hence, it will move faster and will move large distance.

#870161

The number of P - O bonds is P_4O_6 is?

A 9

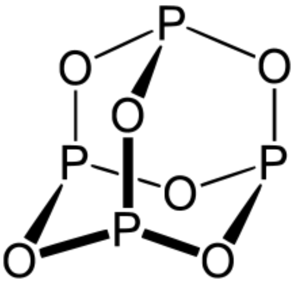
B 6

[✓ **C**] 12

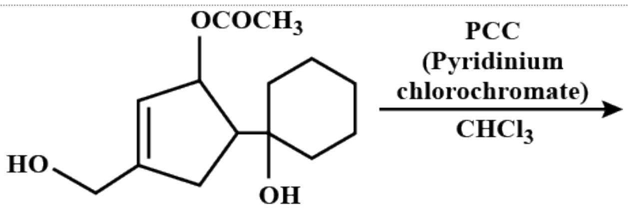
D 18

Solution

The number of P - O bonds is P_4O_6 is 12.

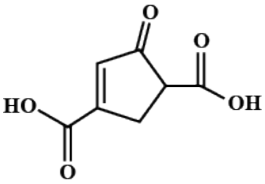


#870162

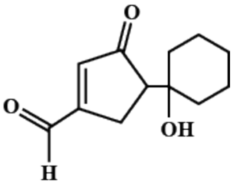


The major product formed in the following reaction is?

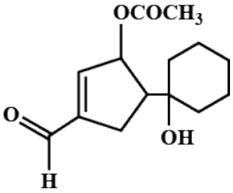
A

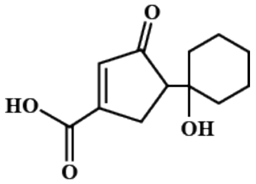


[✓ **B**]



C

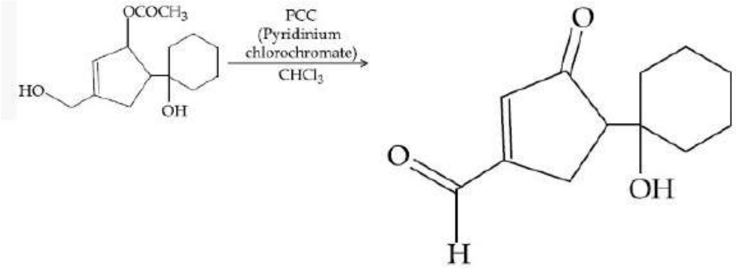


D 

Solution

PCC oxidizes primary alcohols to aldehydes and and secondary alcohols to ketones. PCC does not oxidize aldehydes to carboxylic acids.

In the above reaction, $-OCOCH_3$ group is hydrolyzed to secondary alcohol which is then oxidised (with PCC) to ketone.



#870164

For per gram of reactant, the maximum quantity of N_2 gas is produced in which of the following thermal decomposition reactions? (Given: Atomic wt. $-Cr = 52u, Ba = 137u$).

A $Ba(N_3)_2(s) \rightarrow Ba(C) + 3N_2(g)$

B $(NH_4)_2Cr_2O_7(s) \rightarrow N_2(g) + 4H_2O(g) + Cr_2O_3(s)$

$\boxed{\mathbf{C}}$ $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$

D $2NH_4NO_3(s) \rightarrow 2N_2(g) + 4H_2O(g) + O_2(g)$

Solution

For per gram of reactant, the maximum quantity of N_2 gas is produced in the thermal decomposition $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$

(A)

Molar mass of $Ba(N_3)_2(s) = 221$ g/mol. 1 mole of $Ba(N_3)_2(s)$ will give 3 moles of N_2

$\frac{1g}{221g/mol}$ moles of $Ba(N_3)_2(s)$ will give $3 \times \frac{1}{221} = 0.014$ moles of N_2

(B)

Molar mass of $(NH_4)_2Cr_2O_7 = 252$ g/mol. 1 mole of $(NH_4)_2Cr_2O_7$ will give 1 mole of N_2

$\frac{1g}{252g/mol}$ moles of $(NH_4)_2Cr_2O_7$ will give $1 \times \frac{1}{252} = 0.039$ moles of N_2

(C)

Molar mass of $NH_3 = 17$ g/mol. 2 mole of NH_3 will give 1 mole of N_2

$\frac{1g}{17g/mol}$ moles of NH_3 will give $\frac{1}{2 \times 17} = 0.0297$ moles of N_2

(D)

Molar mass of $NH_4NO_3 = 80$ g/mol. 1 mole of NH_4NO_3 will give 1 mole of N_2

$\frac{1g}{80g/mol}$ moles of NH_4NO_3 will give $1 \times \frac{1}{80} = 0.0125$ moles of N_2

#870169

If x gram of gas is adsorbed by m gram of adsorbent at pressure P, the plot of log $\frac{x}{m}$ versus log P is linear. The slope of the plot is? (n and k are constants and n > 1)

A log k

[✓ B] $\frac{1}{n}$

C 2k

D n

Solution

If x gram of gas is adsorbed by m gram of adsorbent at pressure P, the plot of log $\frac{x}{m}$ versus log P is linear. The slope of the plot is $\frac{1}{n}$ (n and k are constants and n > 1)

According to Freundlich adsorption isotherm,

$$\frac{x}{m} = kP^{\frac{1}{n}}$$

$$log_{10}\frac{x}{m} = \frac{1}{n}log_{10}P + log_{10}k$$

This is the equation of straight line of type $y = mx + c$

$log_{10}k$ is the y intercept.

#870171

Biochemical Oxygen Demand(BOD) value can be a measure of water pollution caused by the organic matter. Which of the following statements is correct?

[✓ A] Polluted water has BOD value higher than 10 ppm

B Aerobic bacteria decrease the BOD value

C Anaerobic bacteria increase the BOD value

D Clean water has BOD value higher than 10 ppm

Solution

Clean water has BOD value less than 5 ppm. Polluted water has BOD value higher than 10 ppm. The option (A) represents correct statement.

#870172

In the leaching method, bauxite ore is digested with a concentrated solution of NaOH that produces 'X'. When CO_2 gas is passed through the aqueous solution of 'X', a hydrated compound 'Y' is precipitated. 'X' and 'Y' respectively are.

A $Na[Al(OH)_4]$ and $Al_2(CO_3)_3 \cdot xH_2O$

B $Al(OH)_3$ and $Al_2O_3 \cdot xH_2O$

C $NaAlO_2$ and $Al_2(CO_3)_3 \cdot xH_2O$

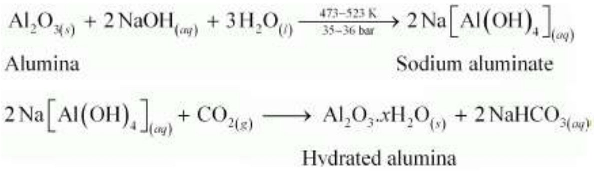
[✓ D] $Na[Al(OH)_4]$ and $Al_2O_3 \cdot xH_2O$

Solution

In the leaching method, bauxite ore is digested with a concentrated solution of NaOH that produces 'X'. When CO_2 gas is passed through the aqueous solution of 'X', a hydrated compound 'Y' is precipitated. 'X' and 'Y' are

$Na[Al(OH)_4]$ and $Al_2O_3 \cdot xH_2O$

respectively.



$$\underset{\text{Alumina}}{Al_2O_{3(s)}} + 2NaOH_{(aq)} + 3H_2O_{(l)} \xrightarrow[35-36\text{ bar}]{473-523\text{ K}} \underset{\text{Sodium aluminate}}{2Na\left[Al(OH)_4\right]_{(aq)}}$$

$$2Na\left[Al(OH)_4\right]_{(aq)} + CO_{2(g)} \longrightarrow \underset{\text{Hydrated alumina}}{Al_2O_3.xH_2O_{(s)}} + 2NaHCO_{3(aq)}$$

#870174

Which of the following statements is not true?

A Chain growth polymerisation involves homopolymerisation only

[✓ B] Chain growth polymerisation includes both homopolymerisation and copolymerisation

C Nylon 6 is an example of step-growth polymerisation

D Step growth polymerisation requires a bifunctional monomer

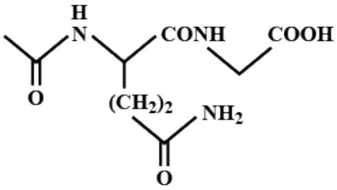
Solution

The statement (B) is not true. Chain growth polymerisation (or addition polymerisation) involves homopolymerisation only. Examples of such polymers include polythene, orlon and teflon.

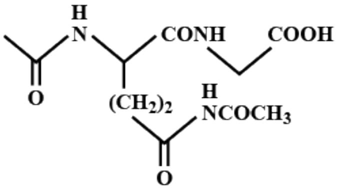
#870175

The dipeptide, Gln-Gly, on treatment with CH_3COCl followed by aqueous work up gives.

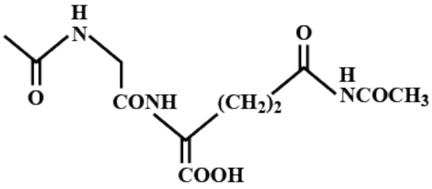
A



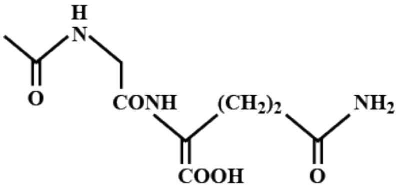
B



C



D



Solution

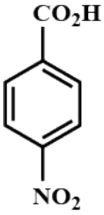
The dipeptide, Gln-Gly, on treatment with CH_3COCl followed by aqueous work up gives the product shown in option (A).

Amino group of glutamine is acetylated. Amide group of glutamine is not acetylated.

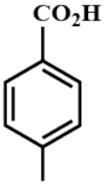
Note:

Acetylation of amide requires activation of amides and/or acyl donors, since the nitrogen atom of amides is less basic than that of the corresponding amines due to amide resonance.

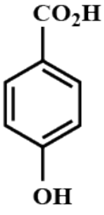
#870176



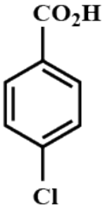
I



II



III



IV

The increasing order of the acidity of the following carboxylic acids is?

A III < II < IV < I

B I < III < II < IV

C IV < II < III < I

D II < IV < III < I

Solution

The increasing order of the acidity of the carboxylic acids is $III < II < IV < I$.

In aromatic acids, electron withdrawing groups like $-Cl,\ -CN,\ -NO_2$ increases the acidity whereas electron releasing groups like $-CH_3,\ -OH,\ -OCH_3,\ -NH_2$ decreases the acidity.

#870142

Let $f : A \to B$ be a function defined as $f(x) = \dfrac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is

A Invertible and $f^{-1}(y) = \dfrac{2y+1}{y-1}$

B Invertible and $f^{-1}(y) = \dfrac{3y-1}{y-1}$

C No invertible

[✓ **D**] Invertible and $f^{-1}(y) = \dfrac{2y-1}{y-1}$

Solution

Let

$y = f(x)$
$\Rightarrow y = \dfrac{x-1}{x-2}$
$\Rightarrow yx - 2y = x - 1$
$\Rightarrow (y-1)x = 2y - 1$
$\Rightarrow x = f^{-1}(y) = \dfrac{2y-1}{y-1}$

So on the given domain the function is invertible and its inverse can be computed as shown above.

So, option D is the correct answer.

#870146

The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to

[✓ **A**] 52

B 44

C 50

D 56

Solution

$(1+x)^2 = 1 + 2x + x^2$
$(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$
$(1+x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$

From the above the binomial expansion, we want the terms containing x^{10} after multiplication

So, the combinations are:

$x.x^9, x.x^6.x^3, x^2.x^2.x^6, x^4.x^6$

Their coefficients are $2 \times 4, 2 \times 1 \times 4, 1 \times 3 \times 6, 3 \times 6$

Which is $8, 8, 18, 18$

Sum of the coefficients is $8 + 8 + 18 + 18 = 52$

Hence, the coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is 52

To find: Coefficeint of

x^{10}

Given, $(1+x)^2(1+x^2)^3(1+x^3)^4$

Using the above series using binomial coefficients,

$\Rightarrow \left[\tbinom{2}{0}x^2 + \tbinom{2}{1}x + \tbinom{2}{2}\right] \times \left[\tbinom{3}{0}x^6 + \tbinom{3}{1}x^4 + \tbinom{3}{2}x^2 + \tbinom{3}{3}\right] \times \left[\tbinom{4}{0}x^{12} + \tbinom{4}{1}x^9 + \tbinom{4}{2}x^6 + \tbinom{4}{3}x^3 + \tbinom{4}{4}\right]$

Finding the coefficient of x^{10}: Multiply individual term to obtain the term x^{10} and note down its coefficients. Find all such possible combinations.

1. $\tbinom{4}{1}x^9 \times \tbinom{3}{3} \times \tbinom{2}{1}x = \tbinom{4}{1} \times \tbinom{3}{3} \times \tbinom{2}{1}x^{10} = 8x^{10}$
2. $\tbinom{4}{2}x^6 \times \tbinom{3}{1}x^4 \times \tbinom{2}{2} = \tbinom{4}{2} \times \tbinom{3}{1} \times \tbinom{2}{2}x^{10} = 18x^{10}$
3. $\tbinom{4}{2}x^6 \times \tbinom{3}{2}x^2 \times \tbinom{2}{0}x^2 = \tbinom{4}{2} \times \tbinom{3}{2} \times \tbinom{2}{0}x^{10} = 18x^{10}$
4. $\tbinom{4}{3}x^3 \times \tbinom{3}{0}x^6 \times \tbinom{2}{1}x = \tbinom{4}{3} \times \tbinom{3}{0} \times \tbinom{2}{1}x^{10} = 8x^{10}$

Adding all the coefficients of x^{10} we get, $52x^{10}$.

Therefore, the coefficient of x^{10} in the expansion is 52.

#870148

If the system of linear equations

$x + ay + z = 3$

$x + 2y + 2z = 6$

$x + 5y + 3z = b$

has no solution, then

A $a = 1, b \neq 9$

B $a \neq -1, b = 9$

C $a = -1, b = 9$

$\boxed{\text{D}}$ $a = -1, b \neq 9$

Solution

If the system of equations has no solution then

$\Delta = 0$ and at least one of Δ_1, Δ_2 and Δ_3 is not zero.

$$\Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$$

$$\implies -a - 1 = 0 \implies a = -1$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 6 \\ 1 & 3 & b \end{vmatrix} \neq 0$$

$$\implies b \neq 9$$

#870150

If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$, and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is

A $-\frac{5}{8}$

B $-\frac{8}{5}$

C $\frac{5}{8}$

$\boxed{\text{D}}$ $\frac{8}{5}$

Solution

Let

α and

$\beta = -1$ be the roots of the polynomial, then we have

$f(x) = x^2 + (1 - \alpha)x - \alpha$.

$f(1) = 2 - 2\alpha$ i

$f(2) = 6 - 3\alpha$ ii

$f(1) + f(2) = 0 \Rightarrow 2 - 2\alpha + 6 - 3\alpha = 0 \Rightarrow \alpha = \frac{8}{5}$

So the other root is $\frac{8}{5}$

So the correct answer is option D.

#870152

The number of four letter words that can be formed using the letters of the word BARRACK is

A 144

B 120

C 264

[✓ **D**] 270

Solution

Case 1: If all four letters are different then the number of words

$={}^{5}C_{4} \times 4! = 120$

Case 2: If 2 letters are R and other 2 different letters are chosen from B, A, C, K then the number of words $={}^{4}C_{2} \times \dfrac{4!}{2!} = 72$

Case 3: If 2 letters are A and other 2 different letters are chosen from B,R,C,K then the number of words $={}^{4}C_{2} \times \dfrac{4!}{2!} = 72$

Case 4: when word is formed using $2R's$ and $2A's$= $\dfrac{4!}{2!2!} = 6$

Then the number of four-letter words that can be formed= $120 + 72 + 72 + 6 = 270$

#870155

The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\dfrac{\pi}{2}, \pi\right)$ is

A 3

B 4

C 2

[✓ **D**] 1

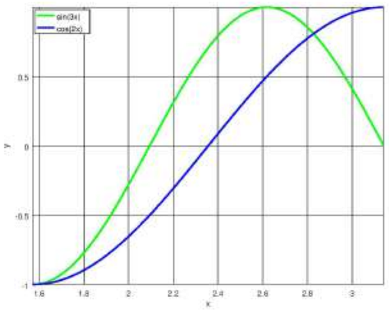
Solution

Let us see the solution graphically as depicted above.

Clearly there is only 1 solution in the given interval.

Note we are asked about the number of solutions and not the solutions themselves, so drawing a graph is enough or one could just check the signs of the function $\sin 3x - \cos 2x$ in this interval and discover that there is only one sign change so there is only one solution. There is no need to solve the equation.

So option D is the correct answer.



#870157

The curve satisfying the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ and passing through the point $(1, 1)$ is

A A circle of radius two

[✓ **B**] A circle of radius one

C A hyperbola

D An ellipse

Solution

$(x^2 - y^2)dx + 2xydy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\Rightarrow \frac{2vdv}{v^2 + 1} = -\frac{dx}{x}$$

Integrating we get;

$ln|v^2 + 1| = -ln|x| + lnc$

$$\frac{y^2}{x^2} + 1 = \frac{c}{x}$$

Putting (1,1)

$c = 2$

$x^2 + y^2 - 2x = 0$

hence its is a circle of radius 1

#870159

A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of $'p'$ is

$\boxed{\text{A}}$ $\frac{1}{3}$

B $\frac{1}{5}$

C $\frac{1}{4}$

D $\frac{2}{5}$

Solution

X wins when the outcome is one of the following set of outcomes:

$H, TTH, TTTTH, \ldots.$

Since subsequent tosses are independent, the probability that X wins is $p + \frac{p}{4} + \frac{p}{16} + \ldots = \frac{4p}{3}$

Similarly Y wins if the outcome is one of the following: $TH, TTTH, TTTTTH, \ldots$

So, the probability that Y wins is $\frac{1-p}{2} + \frac{1-p}{8} + \frac{1-p}{32} = \frac{2(1-p)}{3}$

Since X and Y win with equal probability, we have $\frac{4p}{3} = \frac{2(1-p)}{3} \Rightarrow p = \frac{1}{3}$

So, option A is the correct answer.

#870180

Consider the following two statements:

Statement p:

The value of $\sin 120^\circ$ can be divided by taking $\theta = 240^\circ$ in the equation $2\sin\frac{\theta}{2} = \sqrt{1 + \sin\theta} - \sqrt{1 - 2\theta}$.

Statement q:

The angles A, B, C and D of any quadrilateral $ABCD$ satisfy the equation $\cos\left(\frac{1}{2}(A+C)\right)+\cos\left(\frac{1}{2}(B+D)\right)=0$

Then the truth values of p and q are respectively.

A F, T

B T, T

C F, F

D T, F

Solution

For statement p:

$\sin 120^o = \frac{\sqrt{3}}{2} \Rightarrow 2\sin 120^o = \sqrt{3}$

$\sqrt{1+\sin 240^o} - \sqrt{1-\sin 240^o} = \sqrt{\frac{1-\sqrt{3}}{2}} - \sqrt{\frac{1+\sqrt{3}}{2}} \neq \sqrt{3}$

For statement q:

$\frac{A+C}{2} + \frac{B+D}{2} = \pi \Rightarrow \cos(\frac{A+C}{2}) + \cos(\frac{B+D}{2}) = 0$

So statement p is False and statement q is True. So the correct answer is option A.

#870182

$\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to

A $(-2, -1)$

B $(2, -1)$

C $(-2, 1)$

D $(2, 1)$

Solution

Note that $7-6x-x^2 = 16-(x+3)^2$ and $\frac{d}{dx}(7-6x-x^2) = -2x-6$

So, we have

$\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = \int \frac{2x+6}{\sqrt{7-6x-x^2}}dx - \int \frac{1}{\sqrt{16-(x+3)^2}}dx$

$= -2\sqrt{7-6x-x^2} - \sin^{-1}(\frac{x+3}{4}) + C$

So, we have $A=-2,\ B=-1.$

Thus option A is the correct answer.

#870183

A plane bisects the line segment joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angles. Then this plane also passes through the point.

A $(-3, 2, 1)$

B $(3, 2, 1)$

C $(1, 2, -3)$

D $(-1, 2, 3)$

Solution

Since the Plane BISECTS the line joining the points , then the Plane must meet the line at the Midpoint of the line which is

$(\frac{1-3}{2}, \frac{2+4}{2}, \frac{5+3}{2}) = (\frac{-2}{2}, \frac{6}{2}, \frac{8}{2}) = (-1, 3, 4)$ (As the line is perpendicular to the plane)

Now, the direction cosines of the plane are $(-3-1, 4-2, 5-3) = (-4, 2, 2)$

So the Equation of the Plane must be $= -4x + 2y + 2z = \lambda$

and now since the midpoint of the line is lying in the plane ,it must satisfy the plane

$\therefore -4(-1) + 2(3) + 2(2) = \lambda \Rightarrow \lambda = 18$

Therefore, Equation of plane $\Rightarrow -4x + 2y + 2z = 18$

Now, out of the given option only one point$(-3, 2, 1)$ is satisfying the Plane as follows

$\Rightarrow -4(-3) + 2(2) + 2(1) = 18$

Therefore Correct Answer is A

#870185

If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is

A $\sqrt{13}$

B $\boxed{2\sqrt{13}}$

C 8

D $4 + \sqrt{13}$

Solution

given equation represents the circle with center

$(3, -2)$ and is of radius

$(R) = 4$

$|z|$ represents the distance of point 'z' from origin

Greatest and least distances occur along the normal through the origin

Normal always passes through center of circle

From figure;

let PQ be the normal through origin 'O'

and C be its center $(3, -2)$

it is clear that OP is the least distance

and OQ is the greatest distance

From diagram;

$OP = CP - OC$ and $OQ = CQ + OC$

Here, $CP = CQ = R = 4$

$OC = \sqrt{(3-0)^2 + (-2-0)^2}$

$\Rightarrow OC = \sqrt{13}$

$\therefore OP = CP - OC$

$\Rightarrow OP = 4 - \sqrt{13}$

$\therefore \quad Least \quad distance \quad OP = 4 - \sqrt{13}$

$and \quad OQ = CQ + OC$

$\Rightarrow OQ = 4 + \sqrt{13}$

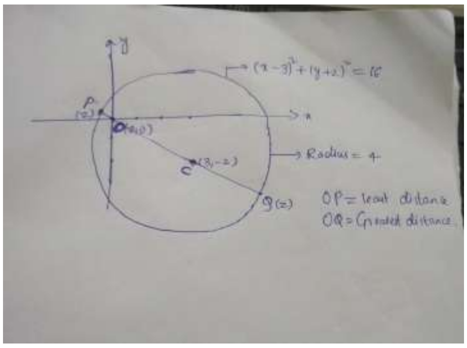
$\therefore \quad Greatest \quad distance = OQ = 4 + \sqrt{13}$

Difference between greatest and least distance=OQ-OP=$\left(4+\sqrt{13}\right)-\left(4-\sqrt{13}\right)$

$\Rightarrow Difference = 2\sqrt{13}$

final answer=$2\sqrt{13}$

the correct option is 'B'



#870187

If the position vectors of the vertices A, B and C of a $\triangle ABC$ are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is

- **A** $\frac{1}{2}(4\hat{i}+8\hat{j}+11\hat{k})$
- [✓ **B**] $\frac{1}{3}(6\hat{i}+13\hat{j}+18\hat{k})$
- **C** $\frac{1}{4}(8\hat{i}+14\hat{j}+9\hat{k})$
- **D** $\frac{1}{3}(6\hat{i}+11\hat{j}+15\hat{k})$

Solution

Let angular bisector of A meets side BC at point P(x, y, z)

By angular bisector theorem we can say that AB:AC=BP:PC

$\therefore BP : PC = c : b$

$\Rightarrow BP : PC = 6 : 3 = 2 : 1 = m : n$

$$\Rightarrow P(x, y, z) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Here B=(2,3,4)=(x_2, y_2, z_2)

and C=(2,5,7)=(x_3, y_3, z_3)

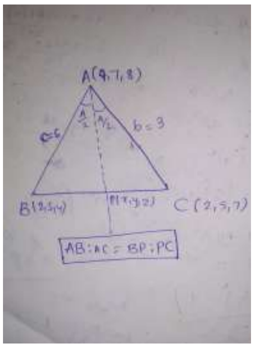
Subtituting values, we get;

$$P(x, y, z) = \left(\frac{(2)(2)+(1)(2)}{2+1}, \frac{(2)(5)+(1)(3)}{2+1}, \frac{(2)(7)+(1)(4)}{2+1}\right)$$

$$P(x, y, z) = \left(\frac{6}{3}, \frac{13}{3}, \frac{18}{3}\right)$$

$\therefore$ Position vector of point $P = \frac{1}{3}(6i + 13j + 18k)$

Hence, the correct option is 'B'.



#870188

The foot of the perpendicular drawn from the origin, on the line, $3x + y = \lambda (\lambda \neq 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio $BP : PA$ is

A $9:1$

B $1:3$

C $1:9$

D $3:1$

Solution

Let (x,y) be foot of perpendicular drawn to the point

(x_1, y_1) on the line

$ax + by + c = 0$

Relation :$\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{-(ax_1 + by_1 + cz_1)}{a^2 + b^2}$

Here (x_1, y_1)=(0,0)

given line is: $3x + y - \lambda = 0$

$$\frac{x - 0}{3} = \frac{y - 0}{1} = \frac{-((3 \times 0) + (1 \times 0) - \lambda)}{3^2 + 1^2}$$

$x = \dfrac{3\lambda}{10}$ and $y = \dfrac{\lambda}{10}$

Hence foot of perpendicular $P = \left(\dfrac{3\lambda}{10}, \dfrac{\lambda}{10}\right)$

Line meets X-axis at $A = \left(\dfrac{\lambda}{3}, 0\right)$

and meets Y-axis at $B = (0, \lambda)$

$$BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2}$$

$$\Rightarrow BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$$

$$\therefore BP = \sqrt{\frac{90\lambda^2}{100}}$$

$$AP = \sqrt{\left(\frac{\lambda}{3} - \frac{3\lambda}{10}\right)^2 + \left(0 - \frac{\lambda}{10}\right)^2}$$

$$\Rightarrow AP = \sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}}$$

$$\therefore AP = \sqrt{\frac{10\lambda^2}{900}}$$

$\therefore BP : AP = 9 : 1$

Hence,correct option is 'A'.

#870189

If $f(x) = \sin^{-1}\left(\dfrac{2 \times 3^x}{1 + 9^x}\right)$, then $f'\left(-\dfrac{1}{2}\right)$ equals.

[✓ **A**] $\sqrt{3}\log_e\sqrt{3}$

B $-\sqrt{3}\log_e\sqrt{3}$

C $-\sqrt{3}\log_e 3$

D $\sqrt{3}\log_e 3$

Solution

Given

$f\,(x) = \sin\left(\dfrac{2 \times 3^x}{1 + 9^x}\right)$

Let $3^x = \tan(t)$

$\Rightarrow f\,(x) = \arcsin\left(\dfrac{2\tan(t)}{1 + \tan^2(t)}\right)$

as $\sin(2t) = \dfrac{2\tan(t)}{1 + \tan^2(t)}$

$\Rightarrow f\,(x) = \arcsin(\sin(2t))$

$\therefore f\,(x) = 2t = 2\arctan(3^x)$

$\rightarrow \dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{2}{1 + (3^x)^2} \times 3^x.\log_e 3$

at x=$\dfrac{1}{2}$ $\rightarrow \dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{2}{1 + \left(3^{\frac{1}{2}}\right)^2} \times 3^{\frac{1}{2}}.\log_e 3$

$= \dfrac{1}{2} \times \sqrt{3} \times \log_e 3$

$= \sqrt{3} \times \log_e\sqrt{3}$

Hence,correct option is 'Ä'.

#870192

Let $A_n = \left(\dfrac{3}{4}\right) - \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3}{4}\right)^3 - \ldots. + (-1)^{n-1}\left(\dfrac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$ is

A 5

[✓ **B**] 7

C 11

D 9

Solution

Formula: Let $a, ar, ar^2 + ar^3 + \ldots + ar^{n-1}$ be n terms of a a GP. Then its sum is given by, $S = \dfrac{a(1 - r^n)}{1 - r}$

Given,

$A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots. + (-1)^{n-1}\left(\frac{3}{4}\right)^n$

It is a Geometric Progression (GP) with $a = \frac{3}{4}, r = \frac{-3}{4}$ and number of terms $= n$

Therefore, $A_n = \frac{\frac{3}{4} \times (1 - (\frac{-3}{4})^n)}{1 - (\frac{-3}{4})}$

$\Rightarrow A_n = \frac{\frac{3}{4} \times (1 - (\frac{-3}{4})^n)}{\frac{7}{4}}$

$\Rightarrow A_n = \frac{3}{7}\left[1 - (\frac{-3}{4})^n\right]$

Also given, $B_n = 1 - A_n$

To find: The least odd natural number p, such that $B_n > A_n$

Now, $1 - A_n > A_n$

$\Rightarrow 1 > 2 \times A_n$

$\Rightarrow A_n < \frac{1}{2}$

Substituting the value of A_n in the above equation, we get

$\frac{3}{7} \times \left[1 - \left(\frac{-3}{4}\right)^n\right] < \frac{1}{2}$

$\Rightarrow 1 - \left(\frac{-3}{4}\right)^n < \frac{7}{6}$

$\Rightarrow 1 - \frac{7}{6} < \left(\frac{-3}{4}\right)^n$

$\Rightarrow \frac{-1}{6} < \left(\frac{-3}{4}\right)^n$

Since n is odd, then $\left(\frac{-3}{4}\right)^n = (-1) \times \frac{3^n}{4}$

Therefore, $\frac{-1}{6} < (-1) \times \left(\frac{3}{4}\right)^n$

Multiplying the entire inequality by -1, we get

$\frac{1}{6} > \left(\frac{3}{4}\right)^n$

Now, Applying log to the base $\frac{3}{4}$

$\log_{\frac{3}{4}} \frac{1}{6} < \frac{3}{4}$

$\Rightarrow 6.228 < n$

Therefore, n should be 7.

#870195

A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram $OABP(O$ being the origin) is formed, then the locus of P is

A $4x^2 - 9y^2 = 121$

B $4x^2 + 9y^2 = 121$

C $9x^2 - 4y^2 = 169$

D $9x^2 + 4y^2 = 169$

Solution

given hyperbola is:

$4x^2 - 9y^2 = 36$

let (x_0, y_0) be point of contact of normal on the hyperbola

Finding slope of normal at that point:

Differntiating hyperbola equation we get; $4.2.x - 9.2.y.\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{4x}{9y}$ = slope of tangent

$\therefore$ slope of normal= $\frac{-9y}{4x}$

equation of normal at (x_0, y_0) is:

$y - y_0 = \frac{-9y_0}{4x_0}(x - x_0)$

line intersects X axis at A when y=0

$\therefore A = \left(\frac{13x_0}{9}, 0\right)$

similarly $B = \left(0, \frac{13y_0}{4}\right)$

given OABP forms a paralleogram→diagonals bisect each other(midpoint of diagonals are same)

midpoint of OB= $\left(0, \frac{13y_0}{8}\right)$ =midpoint of AP

Let P=(x,y)

$\therefore midpointofAP = \left(\frac{\frac{13x_0}{9} + x}{2}, \frac{y}{2}\right)$

$\therefore P(x, y) = \left(\frac{-13x_0}{9}, \frac{13y_0}{4}\right) \rightarrow 1$

As (x_0, y_0) lie on hyperbola,it shoud the its equation:

$4(x_0)^2 - 9(y_0)^2 = 36$

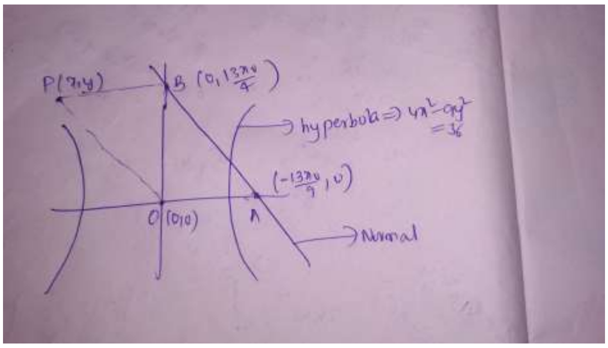
from equation 1: $x_0 = \frac{-9x}{13}$ and $y_0 = \frac{4y}{13}$

substituting in hyperbola equation ,we get:

$9x^2 - 4y^2 = 169$

$\therefore$ locus of point P is hyperbola whose equation is: $9x^2 - 4y^2 = 169$

hence correct option is C.



#870199

Let $f(x)$ be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$.

If $\lim_{x \to 0}\left(\frac{f(x)}{x^2} + 1\right) = 3$ then $f(-1)$ is equal to

A $\frac{1}{2}$

B $\frac{3}{2}$

C $\frac{5}{2}$

$\boxed{\textbf{D}}$ $\frac{9}{2}$

Solution

Given it has extremum values at

$x = 1$ and

$x = 2$

$\Rightarrow f'(1) = 0$ and $f'(2) = 0$

Given f(x) is a fourth degree polynomial

Let $f(x) = ax^4 + bx^3 + cx^2 + dx + c = 0$

Given $\lim_{x \to 0} \left(\frac{f(x)}{x^2} + 1 \right)$=3

$$\lim_{x \to 0} \left(\frac{ax^4 + bx^3 + cx^2 + dx + e}{x^2} + 1 \right) = 3$$

$$\lim_{x \to 0} \left(ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2} + 1 \right) = 3$$

For limit to have finite value (in this case 3) value of 'd' and 'e' must be 0

$\Rightarrow d = 0$ & $e = 0$

Substituting x=0 in limit ;

$\Rightarrow c + 1 = 3$

$\Rightarrow c = 2$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

Applying $f'(1) = 0, f'(2) = 0$

$4a(1) + 3b(1) + 2c(1) + d = 0 \quad \Rightarrow 1$

$4a(8) + 3b(4) + 2c(2) + d = 0 \quad \Rightarrow 2$

Substituting $c = 2$ and $d = 0$

$4a + 3b + 4 = 0$

$32a + 12b + 8 = 0$

Solving two equations, we get $a = \frac{1}{2}$ $and\ b = -2$

$$f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$$

$$f(-1) = \frac{-1^4}{2} - 2(-1)^3 + 2(-1)^2$$

Hence $f(x) = \frac{9}{2}$

Therefore correct option is 'D'

#870202

If the mean of the data : $7, 8, 9, 7, 8, 7, \lambda, 8$ is 8, then the variance of this data is

A $\frac{9}{8}$

B 2

C $\frac{7}{8}$

$\boxed{\textbf{D}}$ 1

Solution

Mean of the date

$7, 8, 9, 7, 8, 7, \lambda, 8$ is

8

$\therefore M = \dfrac{7+8+9+7+8+7+\lambda+8}{8} = 8$

$\Rightarrow \dfrac{54+\lambda}{8} = 8$

$\Rightarrow \lambda = 10$

Now, variance σ^2 is the average of squared difference from mean

$$\text{So, } \sigma^2 = \frac{(7-8)^2+(8-8)^2+(9-8)^2+(7-8)^2+(8-8)^2+(7-8)^2+(10-8)^2+(8-8)^2}{8}$$

$$= \frac{1+0+1+1+0+1+4+0}{8} = \frac{8}{8} = 1$$

Hence, the variance is 1.

#870203

An angle between the lines whose direction cosines are given by the equations, $l+3m+5n=0$ and $5lm-2mn+6nl=0$, is

A $\cos^{-1}\left(\frac{1}{8}\right)$

$\boxed{\textbf{B}}$ $\cos^{-1}\left(\frac{1}{6}\right)$

C $\cos^{-1}\left(\frac{1}{3}\right)$

D $\cos^{-1}\left(\frac{1}{4}\right)$

Solution

Given

$l+3m+5n=0$ ....

1

and $5lm-2mn+6nl=0$ 2

Here l, m, n are directional cosines.

From 1, $l=-3m-5n$

Substituting equation 1 in equation 2

$5(-3m-5n)m-2mn+6n(-3m-5n)=0$

$15m^2+45mn+30n^2=0$

$\Rightarrow m^2+3mn+2n^2=0$

$\Rightarrow m^2+2mn+mn+2n^2=0$

$\Rightarrow (m+n)(m+2n)=0$

$\therefore m=-n$ or $m=-2n$

For $m=-n; l=-2n$

And for $m=-2n; l=n$

$\therefore (l,m,n)=(-2n,-n,n)$

Or $(l,m,n)=(n,-2n,n)$

$\Rightarrow (l,m,n)=(-2,-1,1)$

or $\Rightarrow (l,m,n)=(1,-2,1)$

$cos(\theta) = \frac{A.B}{|A||B|}$, θ is angle between the lines

$\Rightarrow cos(\theta) = \frac{-2.1 + (-1).(-2) + 1.1}{\sqrt{6}.\sqrt{6}}$

$\Rightarrow cos(\theta) = \frac{1}{6}$

$\therefore \theta = \cos^{-1}\left(\frac{1}{6}\right)$

Hence, correct option is 'B'

#870205

The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point $(2, 1)$ cuts off a chord of length 4 from a circle C_2 whose centre is $(3, -2)$. The radius of C_2 is

A $\sqrt{6}$

B 2

C $\sqrt{2}$

D 3

Solution

Equation of tangent on
C_1 at
$(2, 1)$ is:

$2x + y - (x + 2) - 1 = 0$

$x + y = 3$

If it cuts off the chord of the circle C_2 then the equation of the chord is:$x + y = 3$

Distance of the chord from $(3, -2)$

$d = \left|\frac{3 - 2 - 3}{\sqrt{2}}\right| = \sqrt{2}$
Length of the chord is $l = 4$

$r^2 = \frac{l^2}{4 + d^2}$ where r is the radius of the circle.

$r^2 = 4 + 2 = 6$
$\Rightarrow r = \sqrt{6}$

#870206

Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to

A 8

B 12

C 13

D 7

Solution

We have

$$(A-3I)(A-5I)=O$$

$$A^2-8A+15I=O$$

Multiplying both sides with A^{-1}, we get

$$A-8I+15A^{-1}=O$$

$$A+15A^{-1}=8I$$

$$\frac{A}{2}+\frac{15A^{-1}}{2}=4I$$

$$\therefore \alpha+\beta=\frac{1}{2}+\frac{15}{2}=\frac{16}{2}=8$$

#870208

The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$ is

A $\frac{\pi}{2}(\sqrt{2}+1)$

B $\pi(\sqrt{2}-1)$

C $2\pi(\sqrt{2}-1)$

D $\pi\sqrt{2}$

Solution

Given integral is

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$$

As in the denominator there is $1+\sin x$, we first rationalise the denominator and then do integration by parts

So multiplying numerator and denominator by $1-\sin x$, we get

$$\frac{x(1-\sin x)}{1-(\sin x)^2}=\frac{x(1-\sin x)}{(\cos x)^2}$$
$$\Rightarrow = x(1-\sin x)\sec^2 x$$
$$= x\sec^2 x - x\sin x\sec^2 x = x\sec^2 x - x\tan x\sec x$$

Now applying integration by parts to this

$$\int uvdx = u\int vdx - \int \frac{du}{dx}\times\int vdx$$

Therefore by applying the above formula we get

$$I=\int x\sec^2 xdx - \int xsecxtanxdx$$
$$=\left[x\tan x - \int \frac{dx}{dx}tanxdx\right]-\left[x\sec x - \int \frac{dx}{dx}\sec xdx\right]$$
$$=[x\tan x - \ln|\sec x|]-[x\sec x - \ln|\sec x+\tan x|+c$$

Now substituting the limits $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ we get

$$I=\left\{\left[\frac{3\pi}{4}\tan\frac{3\pi}{4}-\ln\left|\frac{3\pi}{4}\right|\right]-\left[\frac{3\pi}{4}\sec\frac{3\pi}{4}-\ln\left|\sec\frac{3\pi}{4}+\tan\frac{3\pi}{4}\right|\right]\right\}-\left\{\left[\frac{\pi}{4}\tan\frac{\pi}{4}-\ln\left|\frac{\pi}{4}\right|\right]-\left[\frac{\pi}{4}\sec\frac{\pi}{4}-\ln|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}|\right]\right\}$$
$$=\frac{\pi}{2}(\sqrt{2}+1)$$

We have,

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x}{1+\sin x}dx$$

Multiply and divide LHS with $(1-\sin x)$, we get

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x(1-\sin x)}{1-\sin^2 x}dx$$

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x(1-\sin x)}{\cos^2 x}dx$$

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}x(\sec^2 x-\sec x\tan x)dx$$

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}x\sec^2 x-\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}x\sec x\tan xdx$$

Using integration by parts

$$I=[(x\tan x)-(\int 1.\tan xdx)]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}+[(x\sec x)-(\int 1.\sec xdx)]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$I=[(x\tan x)-(\log|\sec x|)]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}+[(x\sec x)+(\log|secx+\tan x|]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$I=\frac{\pi}{2}(\sqrt{2}+1)$$

#870212

A tower T_1 of height $60\ m$ is located exactly opposite to a tower T_2 of height $80\ m$ on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is

A $20\sqrt{2}$

B $10\sqrt{2}$

C $10\sqrt{3}$

[✓ **D**] $20\sqrt{3}$

Solution

Let the width of the road between the feet of the towers t_1 and t_2 be w

Let the angles be

$\angle BAC=\theta$ .... [given]
$\Rightarrow \angle EBD=2\theta$

Now, from the above diagram

$$\tan\theta=\frac{opposite\quad side}{hypotenuse}=\frac{BC}{AC}$$
$$\tan 2\theta=\frac{DE}{EB}$$

$BC=80-60=20,$

$AC=w$

$DE=BO=80,$

$EB=DO=w$

$$\therefore \tan 2\theta = \frac{ED}{EB} = \frac{80}{w},$$

$$\tan\theta = \frac{BC}{AC} = \frac{20}{w}$$

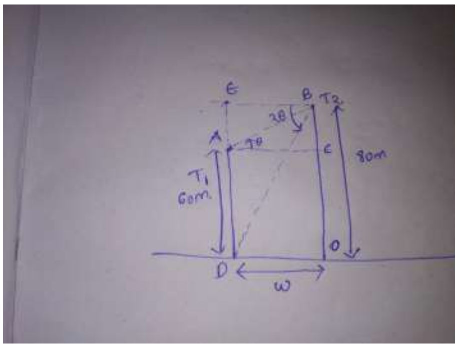
We know that

$$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$$

By substituting the values of $\tan\theta$ and $\tan 2\theta$, we get

$$\frac{80}{w} = \frac{2(\frac{20}{w})}{1-(\frac{20}{W})^2}$$

$$\frac{40}{w} \times w = 80 \times [1-(\frac{20}{w})^2]$$
$$\Rightarrow 40 = 80 \times [1-(\frac{20}{w})^2]$$
$$\Rightarrow [1-(\frac{20}{w})^2] = \frac{40}{80} = \frac{1}{2}$$
$$\Rightarrow 1-\frac{400}{w^2} = \frac{1}{2}$$
$$\Rightarrow \frac{400}{w^2} = \frac{1}{2}$$
$$\Rightarrow 800 = w^2$$
$$\Rightarrow w = \sqrt{800} = 20\sqrt{2}$$



Let the width of the road the road is

d.

If the angle of the elevation is θ, then

$\tan\theta = \frac{20}{x}$, Here 20 is the height difference of T_1 and T_2.

Given that, the angle of depression is twice of the angle of elevation.

$$\tan 2\theta = \frac{60}{d}$$

We know $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$\therefore \frac{60}{d} = \frac{40/d}{1-(400/d^2)}$$

$$\Rightarrow \frac{400}{d^2} = \frac{1}{3}$$

$$d = 20\sqrt{3}$$

#870217

If $I_1 = \int_0^1 e^{-x}\cos^2 x\,dx;\quad I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx$ and $I_3 = \int_0^1 e^{-x^3}dx$; then

A $I_2 > I_3 > I_1$

B $I_3 > I_1 > I_2$

C $I_2 > I_1 > I_3$

[✓ **D**] $I_3 > I_2 > I_1$

Solution

Given:

$I_1 = \int_0^1 e^{-x} \cos^2 x \, dx;$

$I_2 = \int_0^1 e^{-x^2} \cos^2 x \, dx$ and

$I_3 = \int_0^1 e^{-x^3} dx$

For $x \in (0, 1), x > x^2 \Rightarrow -x < -x^2$

$\Rightarrow x^2 > x^3$

$\Rightarrow -x^2 < -x^3$

$\Rightarrow e^{-x^2} < e^{-x^3}$

and $e^{-x} < e^{-x^2}$

$\Rightarrow e^{-x} < e^{-x^2} < e^{-x^3}$

$\Rightarrow e^{-x^3} > e^{-x^2} > e^{-x}$

$\Rightarrow I_3 > I_2 > I_1$

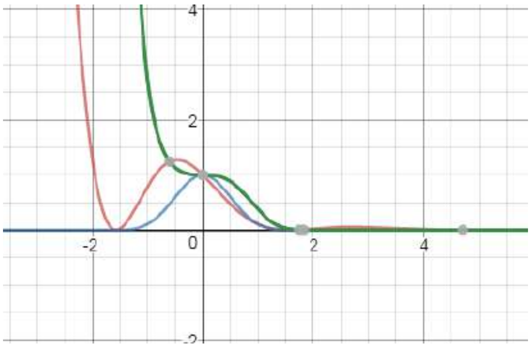
Green line denotes $f(x) = e^{-x} \cos^2 x$

Blue line denotes $g(x) = e^{-x^2} \cos^2 x$

Red line denotes $h(x) = e^{-x^3}$

Also, from the graph we get the same result.

Hence, option D is correct.



#870220

The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is

A 2

B $\frac{7}{4}$

C $\frac{7}{2}$

[✓ **D**] $\frac{5}{2}$

Solution

Let the coordinate A be $(0, c)$

Equations of the parallel lines are given:

$x - y + 2 = 0$ and

$7x - y + 3 = 0$

We know that the diagonals will be parallel to the angle bisectors of the two sides $y = x + 2$ and $y = 7x + 3$

$$\frac{x-y+2}{\sqrt{2}}=\pm\frac{7x-y+3}{5\sqrt{2}}$$

$$5x-5y+10=\pm(7x-y+3)$$

Parallel equations of the diagonals are $2x+4y-7=0$ and $12x-6y+13=0$

slopes of diagonal $m=\frac{-1}{2}$ and 2

We know that the slope of diagonal from $A(0,c)$ and passing through$P(1,2)$ is $(2-c)$

therefore $2-c=2 \implies c=0$, but it is given that A is not origin, so

$$2-c=\frac{-1}{2} \implies c=\frac{5}{2}$$

$\therefore$ coordinate of A is $(0,5/2)$

#870224

$\lim\limits_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ equals.

- **A** 1
- **B** $-\frac{1}{2}$
- **C** $\frac{1}{4}$
- **D** $\frac{1}{2}$

Solution

Given limit is $L=\lim\limits_{x\to 0}\frac{(x\tan 2x-2x\tan x)}{(1-\cos 2x)^2}$

By expanding $\tan 2x$ and $\cos 2x$ we get

$$\frac{(x\tan 2x-2x\tan x)}{(1-\cos 2x)^2}=\frac{x\frac{2\tan x}{1-(\tan x)^2}-2x\tan x}{(1-(1-2\sin^2 x))^2}$$
$$=\frac{2x\tan x-[2x\tan x-2x\tan^3 x]}{4\sin^4 x\times(1-\tan^2 x)}=\frac{2x\tan^3 x}{4\sin^4 x\times(1-\tan^2 x)}$$
$$=\frac{2x\tan^3 x}{4\sin^4 x\times(\frac{\cos^2 x-\sin^2 x}{\cos^2 x})}=\frac{2x\frac{\sin^3 x}{\cos^3 x}}{4\sin^4 x\times(\frac{\cos^2 x-\sin^2 x}{\cos^2 x})}$$
$$=\frac{x}{2\sin x\times(\cos^2 x-\sin^2 x)\cos x}$$

Now applying the limit $x\to 0$, we get

$$L=\lim_{x\to 0}\frac{x}{2\sin x}\times\lim_{x\to 0}\frac{1}{\cos x(\cos^2 x-\sin^2 x)}$$
$$=\lim_{x\to 0}\frac{x}{2\sin x}\times\lim_{x\to 0}\frac{1}{\cos 0(\cos^2 0-\sin^2 0)}$$
$$=\frac{1}{2}[\because\lim_{x\to 0}\frac{x}{\sin x}=1]$$

Hence, option D is correct.

Let us have

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}=l$$

$$l = \lim_{x\to 0} \frac{x\left(\dfrac{2\tan x}{1-\tan^2 x}\right) - 2x\tan x}{\left(1-\dfrac{1-\tan^2 x}{1+\tan^2 x}\right)^2}$$

$$l = \lim_{x\to 0} \frac{2x\tan x\left(\dfrac{1}{1-\tan^2 x} - 1\right)}{\left(\dfrac{2\tan^2 x}{1+\tan^2 x}\right)^2}$$

$$l = \lim_{x\to 0} \frac{2x\tan x\left(\dfrac{\tan^2 x}{1-\tan^2 x}\right)}{\left(\dfrac{2\tan^2 x}{1+\tan^2 x}\right)^2}$$

$$l = \lim_{x\to 0} \frac{x(1+\tan^2 x)^2}{2\tan x(1-\tan^2 x)}$$

$$l = \lim_{x\to 0} \frac{x(\sec^2 x)^2 \cos x}{2\sin x(1-\tan^2 x)}$$

We know $\lim\limits_{x\to 0} \dfrac{x}{\sin x} = 1$

$$\therefore l = \frac{1}{2}$$

#870229

Let $f(x) = \begin{cases} (x-1)\overline{2-x}^{\,1}, x > 1, x \neq 2 \quad k, \\ x = 2 \end{cases}$

The value of k for which f is continuous at $x = 2$ is

A e^{-2}

B e

[✓ **C**] e^{-1}

D 1

Solution

If

$f(x)$ is continuous at

$x = 2$, then

$$\lim_{x\to 2}(x-1)^{\frac{1}{2-x}} = k$$

Above is 1^{∞} form,

$$\therefore k = e^{l}$$

where $l = \lim\limits_{x\to 2}(x-1-1) \times \dfrac{1}{2-x} = \lim\limits_{x\to 2} \dfrac{x-2}{2-x} = -1$

$$\implies k = e^{-1}$$

#870233

If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \dfrac{3}{4}$, then the value of a is

A $\dfrac{1}{4} - \dfrac{1}{3\sqrt{2}}$

B $\frac{1}{4}-\frac{1}{4\sqrt{2}}$

C $\frac{1}{4}-\frac{1}{\sqrt{2}}$

D $\frac{1}{4}-\frac{1}{2\sqrt{2}}$

Solution

If

a, b, c are in A.P. then

$a+c=2b$

Given $a+b+c=\frac{3}{4} \ldots \ldots (1)$

$2b+b=\frac{3}{4} \implies b=\frac{1}{4}$

$b=\frac{1}{4}$

If a^2, b^2, c^2 are in G.P. then

$(b^2)^2=a^2c^2 \implies ac=\pm\frac{1}{16} \ldots \ldots (2)$

From (1) and (2)

$a \pm \frac{1}{16a}=\frac{1}{2}$

$16a^2-8a \pm 1=0$

If $16a^2-8a+1=0 \implies a=\frac{1}{4}$ but it is not true; because $a<b$

If $16a^2-8a-1=0 \implies a=\frac{8 \pm \sqrt{128}}{32}$

$\implies a=\frac{1}{4} \pm \frac{1}{2\sqrt{2}}$

But it is given, that $a<b$

$\therefore a=\frac{1}{4}-\frac{1}{2\sqrt{2}}$

#870237

Tangents drawn from the point $(-8,0)$ to the parabola $y^2=8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to

A 48

B 32

C 24

D 64

Solution

Equation of the chord of contact PQ is given by

$T=0$

$T \equiv 4(x + x_1) - yy_1 = 0$, where $(x_1, y_1) \equiv (-8, 0)$

$\therefore$ chord of contact is $x = 8$

Coordinates of point P and Q are $(8, 8)$ and $(8, -8)$

Focus of the parabola is $F(2, 0)$

Area of triangle $PQF = \frac{1}{2} \times (8 - 2) \times (8 + 8) = 48$ sq. units