#1330202

A uniform metallic wire has a resistance of 18Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is

A 8Ω

B 12Ω

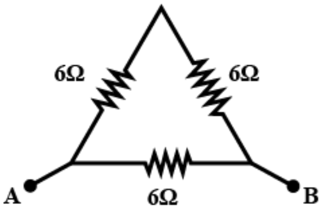
[✓ **C**] 4Ω

D 2Ω

Solution

R_{eq} between any two vertex will be

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{16} \Rightarrow R_{eq.} = 4\Omega.$$



#1330226

A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass $'m'$ is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

A $\frac{3}{2}mv^2$

[✓ **B**] mv^2

C $2mv^2$

D $\frac{1}{2}mv^2$

Solution

At height r from center of earth. orbital velocity

$$= \sqrt{\frac{GM}{r}}$$

$\therefore$ By energy conservation

$$KE \text{ of } 'm' + \left(-\frac{GMm}{r}\right) = 0 + 0$$

(At infinity, $PE = KE = 0$)

$$\Rightarrow KE \text{ of } 'm' = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2.$$

#1330246

A solid metal cube of edge length 2 cm is moving in a positive y direction at a constant speed of $6 m/s$. There is a uniform magnetic field of $0.1\ T$ in the positive $z-direction$. The potential difference between the two faces of the cube perpendicular to the $x-axis$, is:

A $6\ mV$

B $1\ mV$

[✓ **C**] $12\ mV$

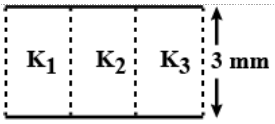
D $2\ mV$

Solution

Potential difference between two faces perpendicular to x-axis will be

$l.(\vec{V} \times \vec{B}) = 12\ mV.$

#1330281



A parallel plate capacitor is of area $6\ cm^2$ and a separation $3\ mm$. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constant $K_1, = 10$, $K_2 = 12$ and,$K_3 + 14$. The dielectric constant of a material which when fully inserted in above capacitor gives same capacitance would be

$\boxed{\text{A}}$ 12

B 4

C 36

D 14

Solution

Let dielectric constant of material used be K.

$$\therefore \frac{10\epsilon_0 A/3}{d} + \frac{12\epsilon_0 A/3}{d} + \frac{14\epsilon_0 A/3}{d} = \frac{K\epsilon_0 A}{d}$$

$\Rightarrow K = 12.$

Let dielectric constant of material used be K.

$$\therefore \frac{10\epsilon_0 A/3}{d} + \frac{12\epsilon_0 A/3}{d} + \frac{14\epsilon_0 A/3}{d} = \frac{K\epsilon_0 A}{d}$$

$\Rightarrow K = 12.$

#1330297

A $2\ W$ carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is

A 63 mA

B 0.4 mA

C 100 mA

$\boxed{\text{D}}$ 20 mA

Solution

$P = i^2R.$

$\therefore$ for i_{max}, R must be minimum

from color coding $R = 50 \times 10^2\Omega$

$\therefore i_{max} = 20\ mA.$

#1330328

In a Young's double slit experiment with slit separation $0.1\ mm$, one observes a bright fringe at angle $\frac{1}{40} rad$ by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range ($380\ nm$ to $740\ nm$), their values are

A 380 nm, 500 nm

$\boxed{\text{B}}$ 625 nm, 500 nm

C 380 nm, 525 nm

D 400 nm, 500 nm

Solution

Path difference $= d\sin\theta = d\theta$

$= 0.1 \times \frac{1}{40} mm = 2500\ nm$

or bright fringe, path difference must be integral multiple of λ.

$\therefore 2500 = n\lambda_1 = m\lambda_2$

$\therefore \lambda_1 = 625, \lambda_2 = 500$ (from $m = 5$)

(for $n = 4$).

#1330345

A magnet of total magnetic moment $10^{-2}\hat{i}\ A - m^2$ is placed in a time varying magnetic field. $B\hat{i}(\cos\omega t)$ where $B = 1$ Tesla and $\omega = 0.125\ rad/s$. The work done for reversing the direction of the magnetic moment at $t = 1$ second, is___?

A 0.007 J

[✓ **B**] 0.02 J

C 0.01 J

D 0.028 J

Solution

Work done, $W = (\Delta\vec{\mu}) \cdot \vec{B}$

$= 2 \times 10^{-2} \times 1\cos(0.125)$

$= 0.02\ J$

$\therefore$ correct answer is (2).

#1330369

To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is

[✓ **A**] $\frac{2}{3}\mu FR$

B $\mu FR/3$

C $\mu FR/2$

D $\mu FR/6$

Solution

Consider a strip of radius x and thickness dx,

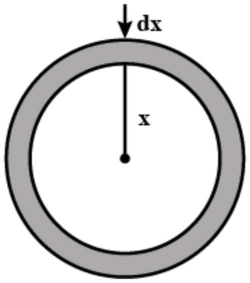
Torque due to friction on this strip.

$$\int d\tau = \int_0^R \frac{x\mu F \cdot 2\pi x dx}{\pi R^2}$$

$$\tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

$\therefore$ correct answer is (1).



#1330412

Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t = 0$ it was 1600 counts per second and $t = 8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t = 6$ seconds is close to

A 150

B 360

[✓ **C**] 200

D 400

Solution

At $t = 0, A_0 = \frac{dN}{dt} = 1600\ C/s$

at $t = 8s, A = 100\ C/s$

$\frac{A}{A_0} = \frac{1}{16}$ in 8 sec

Therefore half life is $t_{1/2} = 2\ sec$

$\therefore$ Activity at $t = 6$ will be $1600\left(\frac{1}{2}\right)^3$

$= 200\ C/s$

$\therefore$ correct answer is (3).

#1330431

If the magnetic field of a plane electromagnetic wave is given by (The speed of light $= 3 \times 10^8 / m/s) B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is

A $4 \times 10^4 N/C$

B $4.5 \times 10^4 N/C$

C $6 \times 10^4 N/C$

[✓ **D**] $3 \times 10^4 N/C$

Solution

$E_0 = B_0 \times c$

$= 100 \times 10^{-6} \times 3 \times 10^{8}$

$= 3 \times 10^{4} N/C$

$\therefore$ correct answer is $3 \times 10^{4} N/C$.

#1330495

A charge Q is distributed over three concentric spherical shells of radii $a, b, c (a < b < c)$ such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where $r < a$, would be

A $\dfrac{Q}{4\pi\epsilon_0(a+b+c)}$

$\boxed{\text{B}}$ $\dfrac{Q(a+b+c)}{4\pi\epsilon_0(a^2+b^2+c^2)}$

C $\dfrac{Q}{12\pi\epsilon_0}\dfrac{ab+bc+ca}{abc}$

D $\dfrac{Q}{4\pi\epsilon_0}\dfrac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$

Solution

Potential at point P, $V = \dfrac{kQ_a}{a} + \dfrac{kQ_b}{b} + \dfrac{kQ_c}{c}$

$\because Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$

$\{since\ \sigma_a = \sigma_b = \sigma_c\}$

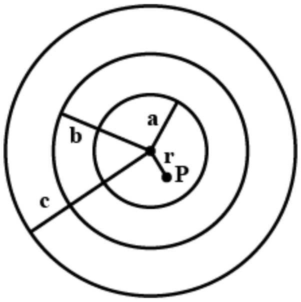
$$\therefore Q_a = \left[\frac{a^2}{a^2+b^2+c^2}\right]Q$$

$$Q_b = \left[\frac{b^2}{a^2+b^2+c^2}\right]Q$$

$$Q_c = \left[\frac{c^2}{a^2+b^2+c^2}\right]Q$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{(a+b+c)}{a^2+b^2+c^2}\right]$$

$\therefore$ correct answer is (2).



#1330516

Water flows into a large tank with flat bottom at the rate of $10^{-4} m^3 s^{-1}$. Water is also leaking out of a hole of area $1\ cm^2$ at its bottom. If the height of the water in the tank remains steady, then this height is__?

A $4\ cm$

B $2.9\ cm$

C $1.7\ cm$

$\boxed{\text{D}}$ $5.1\ cm$

Solution

Since height of water column is constant therefor, water inflow rate (Q_{in})

$=$ water outflow rate

$Q_{in} = 10^{-4} m^3 s^{-1}$

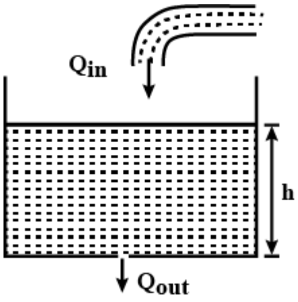
$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4} = \sqrt{20 \times h}$

$h = \frac{1}{20} m$

$h = 5\ cm$

$\therefore$ correct answer is (4).



#1330574

A piece of wood of mass $0.03\ kg$ is dropped from the top of a $100\ m$ height building. At the same time, a bullet of mass $0.02\ kg$ is fired vertically upward, with a velocity $100\ ms^{-1}$, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ($g = 10\ ms^{-2}$).

A 30 m

B 10 m

[✓ **C**] 40 m

D 20 m

Solution

Time taken for the particles to collide,

$t = \frac{d}{V_{rel}} = \frac{100}{100} = 1\ sec$

Speed of wood just before collision $= gt = 10\ m/s$ and speed of bullet just before collision $v - gt = 100 - 10 = 90\ m/s$

Now, conservation of linear momentum just before and after the collision -

$-(0.02)(1v) + (0.02)(9v) = (0.05)v$

$\Rightarrow 150 = 5v$

$\Rightarrow v = 30\ m/s$

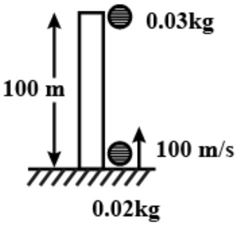
Max. height reached by body $h = \frac{v^2}{2g}$

Before : $0.03\ kg \downarrow 10\ m/s$

$0.02\ kg \uparrow 90\ m/s$

After : $v\ 0.05\ kg$

$h = \frac{30 \times 30}{2 \times 10} = 40m.$



#1330591

The density of a material in SI units is $128\ kg\ m^{-3}$. In certain units in which the unit of length is $25\ cm$ and the unit of mass is $50\ g$, the numerical value of density of the material is

A 410

B 640

C 16

$\boxed{\text{D}}$ 40

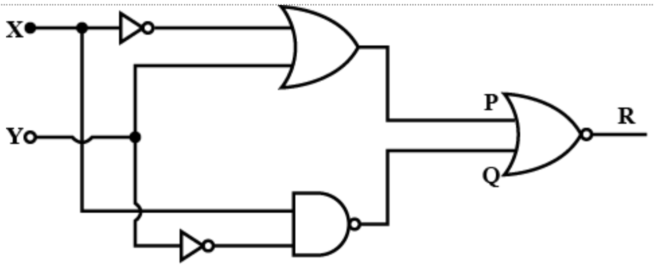
Solution

$$\frac{128\ kg}{m^3} = \frac{125(50\ g)(20)}{(25\ cm)^4(4)^3}$$

$$= \frac{128}{64}(20)units$$

$$= 40\ units.$$

#1330615



To get output $'1'$ and R, for the given logic gate circuit the input values must be

A $X = 0,\ Y = 1$

B $X = 1,\ Y = 1$

C $X = 0,\ Y = 0$

$\boxed{\text{D}}$ $X = 1,\ Y = 0$

Solution

$p = \bar{x} + y$

$Q = \overline{\bar{y} \cdot x} = y + \bar{x}$

$O/P = \overline{P + Q}$

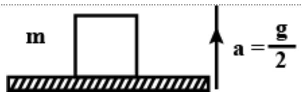
To make O/P

$P + Q$ must be $'O'$

So, $y = 0$

$x = 1.$

#1330639



A block of mass m is kept on a platform which starts from rest with a constant acceleration $g/2$ upwards, as shown in the figure. Work done by normal reaction on block in time t is____?

A 0

$\boxed{\text{B}}$ $\dfrac{3mg^2t^2}{8}$

C $-\frac{mg^2t^2}{8}$

D $\frac{mg^2t^2}{8}$

Solution

$N - mg = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

Now, work done $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}gt^2\right)$

$\Rightarrow W = \frac{3mg^2t^2}{4}$.

#1330667

A heat source at $T = 10^3 K$ is connected to another heat reservoir at $T = 10^2 K$ by a copper slab which is $1\ m$ thick. Given that the thermal conductivity of copper is $0.1\ KW^{-1}m^{-1}$, the energy flux through it in the steady state is

A 90 Wm^{-2}

B 200 Wm^{-2}

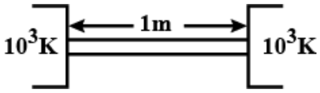
C 65 Wm^{-2}

D 120 Wm^{-2}

Solution

$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{l}$

$\Rightarrow \frac{1}{a}\left(\frac{dQ}{dt}\right) = \frac{(0.1)(900)}{1} = 90\ Wm^2$.



#1330704

A TV transmission tower has a height of $140\ m$ and the height of the receiving antenna is $40\ m$. What is the maximum distance upto which signals can be broad casted from this tower in LOS(Line of Sight) mode? (Given : radius of earth $= 6.4 \times 10^6 m$).

A 80 km

B 48 km

C 40 km

D 65 km

Solution

Maximum distance upto which signal can be broadcasted is

$d_{max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

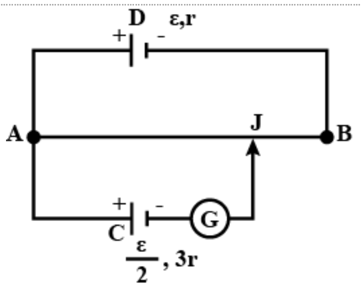
where h_R and h_R are heights of transmitter tower and height of receiver respectively.

Putting all values -

$d_{max} = \sqrt{2 \times 6.4 \times 106}[\sqrt{104} + \sqrt{40}]$

on solving, $d_{max} = 65\ km$.

#1330736



A potentiometer wire AB having length L and resistance $12\ r$ is joined to a cell D of emf ϵ and internal resistance r. A cell C having emf $\epsilon/2$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is

A $\frac{5}{12}L$

B $\frac{11}{24}L$

C $\frac{11}{12}L$

[✓ D] $\frac{13}{24}L$

Solution

$i = \frac{\epsilon}{13r}$

$i\left(\frac{x}{L}12r\right) = \frac{\epsilon}{2}$

$\frac{\epsilon}{13}\left[\frac{x}{L} \cdot 12r\right] = \frac{\epsilon}{2} \Rightarrow x = \frac{13L}{24}.$

#1330783

An insulating thin rod of length l as a x linear charge density $p(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin $(x = 0)$ and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is

[✓ A] $\frac{\pi}{4}n\rho l^3$

B $n\rho l^3$

C $\pi n\rho l^3$

D $\frac{\pi}{3}n\rho l^3$

Solution

$\because M = NIA$

$dq = \lambda dx$ and $A = \pi x^2$

$\int dm = \int(x) = \frac{\rho_0 x}{l}dx.\ \pi x^2$

$M = \frac{n\rho_0\pi}{l} \cdot \int_0^l x^3 \cdot dx = \frac{n\rho_0\pi}{l} \cdot \left[\frac{L^4}{4}\right]$

$M = \frac{n\rho_0\pi l^3}{4}$ or $\frac{\pi}{4}n\rho l^3.$

#1330803

Two guns A and B can fire bullets at speeds $1\ km/s$ and $2\ km/s$ respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns,on the ground is

A $1:2$

B $1:4$

C 1:8

D 1:16

Solution

$R = \dfrac{u^2 \sin 2\theta}{g}$

$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$\dfrac{A_1}{A_2} = \dfrac{u_1^4}{u_2^4} = \left[\dfrac{1}{2}\right]^4 = \dfrac{1}{16}.$

#1330842

A string of length $1\ m$ and mass $5\ g$ is fixed at both ends. The tension in the string is $8.0\ N$. The siring is set into vibration using an external vibrator of frequency $100\ Hz$. The separation between successive nodes on the string is close to__?

A 16.6 cm

B 20.0 cm

C 10.0 cm

D 33.3 cm

Solution

Velocity of wave on string

$V = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{8}{5} \times 1000} = 40\ m/s$

Now, wavelength of wave $\lambda = \dfrac{v}{n} = \dfrac{40}{100} m$

Separation b/w successive nodes, $\dfrac{\lambda}{2} = \dfrac{20}{100} m$

$= 20\ cm.$

#1330883

A train moves towards a stationary observer with speed $34\ m/s$. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to $17\ m/s$, the frequency registered is f_2. If speed of sound is $340\ m/s$, then the ratio f_1/f_2 is__?

A 18/17

B 19/18

C 20/19

D 21/20

Solution

$f_{app} = f_0\left[\dfrac{v_2 \pm v_0}{v_2 \mp v_s}\right]$

$f_1 = f_0\left[\dfrac{340}{340 - 34}\right]$

$f_2 = f_0\left[\dfrac{340}{340 - 17}\right]$

$\dfrac{f_1}{f_2} = \dfrac{340 - 17}{340 - 34} = \dfrac{323}{306} \Rightarrow \dfrac{f_1}{f_2} = \dfrac{19}{18}.$

#1330915

In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of $7.5 \times 10^{-12} m$, the minimum electron energy required is close to

A 100 keV

B 500 keV

[✓ **C**] 25 keV

D 1 keV

Solution

$\lambda = \frac{h}{p}\left\{\lambda = 7.5 \times 10^{-12}\right\}$

$P = \frac{h}{\lambda}$

$$KE = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left\{\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right\}}{2 \times 9.1 \times 10^{-31}}$$

$KE = 25\ KeV$.

#1330933

A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is____?

A $\frac{3F}{2mR}$

B $\frac{F}{3mR}$

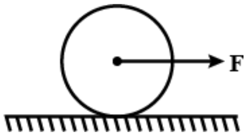
[✓ **C**] $\frac{2F}{3mR}$

D $\frac{F}{2mR}$

Solution

$FR = \frac{3}{2}MR^2\alpha$

$\alpha = \frac{2F}{3MR}$.



#1330964

A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as

A $\mu_1 + \mu_2 = 3$

[✓ **B**] $2\mu_1 - \mu_2 = 1$

C $2\mu_2 - \mu_1 = 1$

D $3\mu_2 - \mu_1 = 1$

Solution

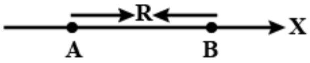
$$\frac{1}{2f_2} = \frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{(\mu_1 - 1)}{R} = \frac{(\mu_2 - 1)}{2R}$$

$$2\mu_2 - \mu_2 = 1.$$

#1330993



Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ placed on the x-axis with a separation R, as shown in the figure.

The distance from A at which both of them produce the same potential is

A $\frac{\sqrt{2}R}{\sqrt{2}+1}$

B $\frac{R}{\sqrt{2}+1}$

$\boxed{\text{C}}$ $\frac{\sqrt{2}R}{\sqrt{2}-1}$

D $\frac{R}{\sqrt{2}-1}$

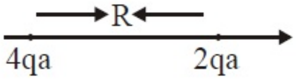
Solution

$$V = \frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$$

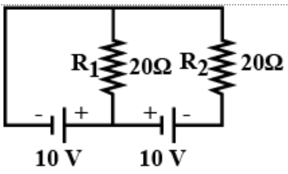
$$\sqrt{2}x = R + x$$

$$x = \frac{R}{\sqrt{2}-1}$$

$$dist = \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}.$$



#1331021



In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1, and R_2 respectively, are_____?

A 2, 2

B 0, 1

C 1, 2

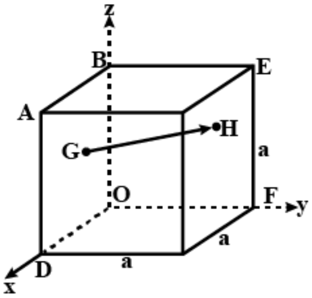
$\boxed{\text{D}}$ 0.5, 0

Solution

$$i_1 = \frac{10}{20} = 0.5A$$

$$i_2 = 0.$$

#1331040



In the cube of side $'a'$ shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be

A $\frac{1}{2}a(\hat{i} - \hat{k})$

[✓ B] $\frac{1}{2}a(\hat{j} - \hat{i})$

C $\frac{1}{2}a(\hat{k} - \hat{i})$

D $\frac{1}{2}a(\hat{j} - \hat{k})$

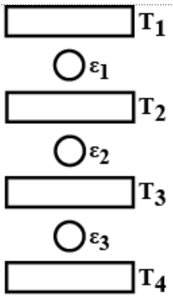
Solution

$\vec{r}_g = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}$

$\vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$\vec{r}_H - \vec{r}_g = \frac{a}{2}(\hat{j} - \hat{i})$.

#1331085



Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2, and T_3, as shown, with $T_2 > T_2 > T_3 > T_4$. The three engines are equally efficient if__?

[✓ A] $T_2 = (T_1^2 T_4)^{1/3}: T_3 = (T_1 T_4^2)^{1/3}$

B $T_2 = (T_1 T_4^2)^{1/3}: T_3 = (T_1^2 T_4)^{1/3}$

C $T_2 = (T_1^3 T_4)^{1/4}: T_3 = (T_1 T_4^3)^{1/4}$

D $T_2 = (T_1 T_4)^{1/2}: T_3 = (T_1^2 T_4)^{1/3}$

Solution

$t_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{T_2} = 1 - \frac{T_4}{T_3}$

$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_4}{T_3}$

$\Rightarrow T_2 = \sqrt{T_1 T_3} = \sqrt{T_1 \sqrt{T_2 T_4}}$

$T_3 = \sqrt{T_2 T_4}$

$T_2^{3/4} = \sqrt{T_1^{1/2} T_4^{1/4}}$

$T_2 = T_1^{2/3} T_4^{1/3}$.

#1329237

Two π and half σ bonds are present in:

[✓ A] N_2^+

B N_2

C O_2^+

D O_2

Solution

$N_2^{\oplus} \Longrightarrow BO = 2.5 \Longrightarrow \left[\pi - Bond = 2\&\sigma - Bond = \dfrac{1}{2}\right]$

$N_2 \Longrightarrow B.O. = 3.0 \Longrightarrow [\pi - Bond = 2\&\sigma - Bond = 11]$

$O_2^{\oplus} = B.O. \Longrightarrow 2.5 \Longrightarrow [\pi - Bond = 1.5\&\sigma - Bond = 11]$

$O_2 \Longrightarrow 2 \Longrightarrow [\pi - Bond \Longrightarrow 1\&\sigma - Bond = 11]$

#1329275

The chemical nature of hydrogen preoxide is:

A oxidising and reducing agent in acidic medium, but not in basic medium.

[✓ B] oxidising and reducing agent in both acidic and basic medium

C reducing agent in basic medium, but not in acidic medium

D oxidising agent in acidic medium, but not in basic medium.

Solution

H_2O_2 act as oxidisong agent and reducing agent in acidic medium as well as basic medium.

H_2O_2 Act as oxidant:

$H_2O_2 + 2H^{\oplus} + 2e^{\ominus} \rightarrow 2H_2O$(In acidic medium)

$H_2O + 2e^{\ominus} \rightarrow 2OH^{\ominus}$ (In basic medium)

H_2O_2 Acts as reductant:-

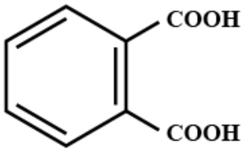
$H_2O_2 \rightarrow 2H^+ + O_2 + 2e^{\ominus}$ (In acidic medium)

$H_2O_2 + 2OH^{\ominus} \rightarrow 2H_2O + O_2 + 2e^{\ominus}$(in basic medium)

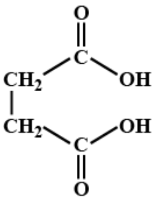
#1329341

Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride?

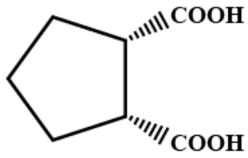
A



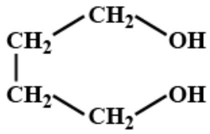
B



C



D



Solution

Adipic acid $CO_2H-(CH_2)_4-CO_2H \xrightarrow[agent]{dehydrating} 7$ membered cyclic anhydride (Very unstable).

#1329356

Which premitive unit cell has unequal edge lengths ($a \neq b \neq c$) and all axial angles different from 90?

A Tetragonal

B Hexagonal

C Monoclinic

D Triclinic

Solution

In Triclinic cell

$a \neq b \neq c \& \alpha \neq \beta \neq \gamma \neq 90^o$.

#1329376

Wilkinson catalyst is:

A $[(Ph_3P)_3RhCl]\ (Et = C_2H_5)$

B $[Et_3P)_3IrCl]$

C $[Et_3P)_3RhCl]$

D $[Ph_3P)_3IeCl]$

Solution

Wilkinsion catalyst is $[(Ph_3P)_3RhCl]$

#1329406

The total number of isotopes of hydrogen and number of radioactive iostopes among them, respectively, are:

A 2 and 0

B 3 and 2

C 3 and 1

D 2 and 1

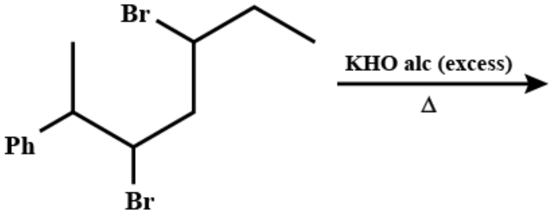
Solution

Total number of isotopes of hydrogen is 3

$\Longrightarrow {}_{2}^{1}H, {}_{1}^{2}H$ or ${}_{1}^{3}D, {}_{1}^{3}H$ or ${}_{1}^{3}T$

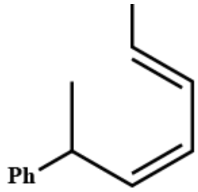
and only ${}_{1}^{3}H$ or ${}_{1}^{3}T$ is an Radioactive element.

#1329439

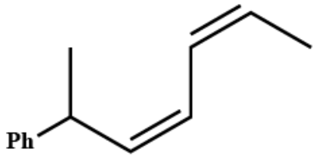


The major product of the following reaction is:

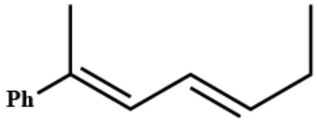
A



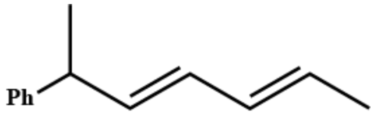
B



$\boxed{\text{C}}$



D



Solution

Example of E_2 elimination and conjugated diene is formed with phenyl ring in conjugation which makes it very stable.

#1329456

The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$:

$\boxed{\text{A}}$ 12

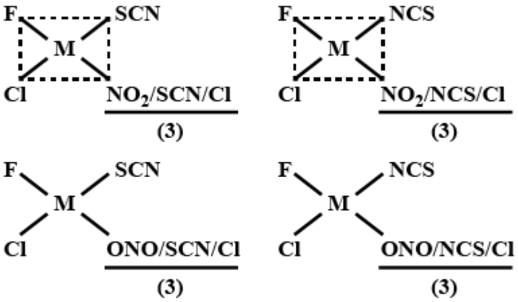
B 8

C 16

D 4

Solution

The total number of isomer for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is 12.



#1329666

Hall-Heroult's process is given by:

A $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

B $Cu^{2+}(aq) + H_2(g) \rightarrow Cu(s) + 2H^+(aq)$

C $ZnO + C \xrightarrow{Coke, 1673K} Zn + CO$

[✓ D] $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Solution

In Hall-Heroult's process is given by

$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

$2Al_2O_3(\ell) \rightleftharpoons 4Al^{3+}(\ell) + 6O^{2\ominus}(\ell)$

At cathode: $4Al^{3+}_{(\ell)} + 12e^{\ominus} \rightarrow 4Al(\ell)$

At Anode: $6O^{2\ominus}_{(\ell)} \rightarrow 2O_2(g) + 12e^{\ominus}$

$3C + 3O_2 \rightarrow 3CO_2(\uparrow)$

#1329701

The value of $\frac{K_p}{K_c}$ for the following reactions at $300K$ are, respectively:

$(At\ 300K, RT = 24.62\, dm^3\, atm\, mol^{-1})$

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

A $1, 24.61\, dm^3\, atm\, mol^{-1}, 606.0\, dm^6\, atm^2\, mol^{-2}$

B $1, 4.1 \times 10^{-2}\, dm^{-3}\, sm^{-1}\, mol^{-1}, 606.0\, dm^6\, atm^2\, mol^{-2}$

C $606.0\, dm^6\, atn^6\, mol^{-2}, 1.65 \times 10^{-3}\, dm^3\, atm^{-2}\, mol^{-1}$

[✓ D] $1, 24.62\, dm^3\, atm\, mol^{-1}, 1.65 \times 10^{-3}\, dm^{-6}\, atm^{-2}\, mol^2$

Solution

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$$\frac{k_p}{k_c} = (RT)^{\Delta n_g} = (RT)^o = 1$$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$$\frac{k_p}{k_c} = (RT)^1 = 24.62$$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$$\frac{k_p}{k_c} = (RT)^{-2} = \frac{1}{(RT)^2} = 1.65 \times 10^{-3}$$

#1329781

If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct ?

[✓ A] DCM and H_2O would stay as lower and upper layer respectively in the S.F.

B DCM and H_2O will be miscible clearly

C DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)

D DCM and H_2O will make trubid/colloidal mixture

Solution

All Alkyl Halides are absolutely water insoluble.

All Alkyl Halides are dense as compared to water.

In a separating funnel upper layer is called layer-1 and the lower layer is called layer-2.

So the upper layer will be with less dense H_2O and the lower layer will be denser DCM.

Hence option A is a correct answer.

#1329795

The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$,respectively, are:

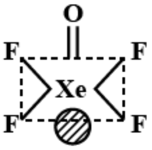
A sp^3d and 1

B sp^3d and 2

[✓ C] sp^3d^2 and 1

D sp^3d^2 and 2

Solution



$sp^3d^2 \Rightarrow [5\sigma\text{-bond} + 1 \text{ l.p.}]$

#1329814

The metal used for making X-ray window is:

A Mg

B Na

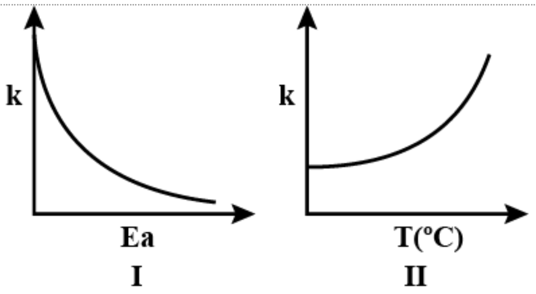
C Ca

[✓ D] Be

Solution

Be Metal is used in x-ray window is due to transparent to x-rays.

#1329844



Consider the given plots for a reaction obeying Arrhenius equation $(0^oC < T < 300^oC)$: (k and E_a are rate constant and activation energy, respectively)

Choose the correct option.

A Both I and II are wrong

B I is wrong but II is right

C Both I and II are correct

[✓ D] I is right but II is wrong

Solution

On increasing E_a, K decreases.

#1329860

Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is:

A A is more polluted than B

B A is suitable for drinking, whereas B is not

[✓ C] B is more polluted that A

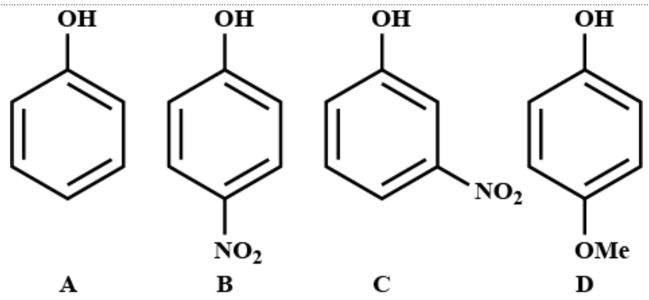
D Both A and B are suitable for drinking

Solution

Two glasses "A" and "B" have BOD values 10 and "20", respectively.

Hence glasses "B" is more polluted than glasses "A".

#1329883



The increasing order of the pKa values of the following compounds is:

A $D < A < C < B$

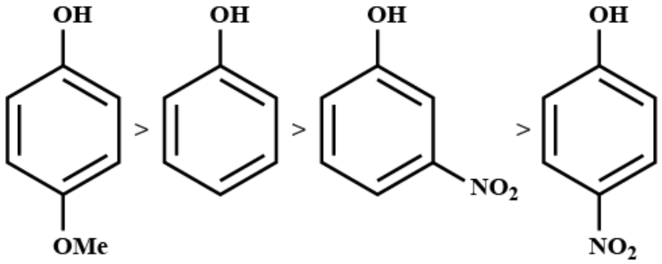
B $B < C < D < A$

C $C < B < A < D$

[✓ D] $B < C < A < D$

Solution

Acidic strength is inversely proportional to pKa.



#1329905

Liquids A and B form an ideal solution in the entire composition range. At $350K$, the vapor pressures of pure A and pure B are $7 \times 10^3\ Pa$ and $12 \times 10^3\ Pa$, respectively.

The composition of the vapor in equilibrium with a solution containing 40 mole percent of A at this temperature is:

A $x_A = 0.37, x_B = 0.63$

$\boxed{B}$ $x_A = 0.28; x_B = 0.72$

C $x_A = 0.76; x_B = 0.24$

D $x_A = 0.4; x_B = 0.6$

Solution

$$y_A = \frac{P_A}{P_{Total}} = \frac{P_A^o x_A}{P_A^o x_A \times P_B^o x_B}$$

$$= \frac{7 \times 10^3 \times 0.4}{7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6}$$

$$= \frac{2.8}{10} = 0.28$$

$y_B = 0.72.$

#1329969

Consider the following processes:

$Zn^{2+} + 2r^- \rightarrow Zn(s); E^o = -0.76\,A$

$Ca^{2+} + 2e^- \rightarrow Ca(s); E^o = -2.87\,V$

$Mg^{2+} + 2e^- \rightarrow Mg(s); E^o = -2.36\,V$

$Ni^{2+} + 2e^- \rightarrow Ni(s); E^o = -0.25\,V$

The reducing power of the metals increases in the order:

A $Ca < Zn < Mg < Ni$

$\boxed{B}$ $Ni < Zn < Mg < Ca$

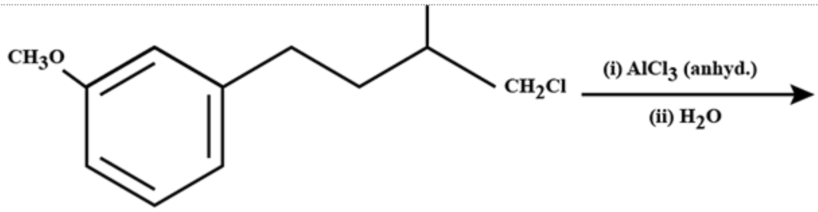
C $Zn < Mg < Ni < Ca$

D $Ca < Mg < Zn < Ni$

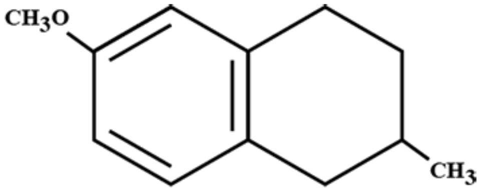
Solution

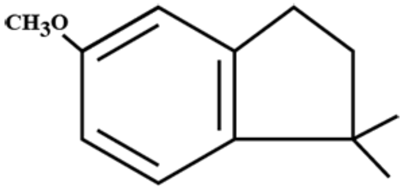
Higher the oxidation potential better will be reducing power.

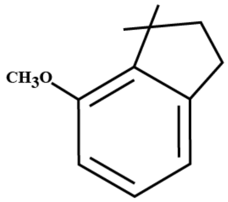
#1329995

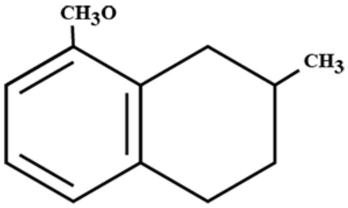


The major product of the following reaction is:

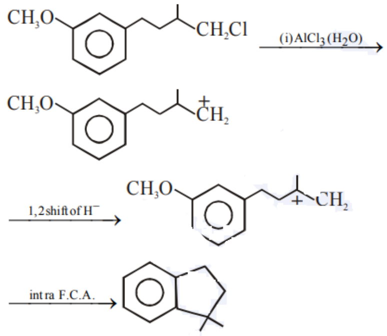
A


$\boxed{B}$


C 

D 

Solution



#1329999

The electronegativity of aluminium is similar to:

A Boron

B Carbon

C Lithium

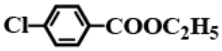
[✓ D] Berylium

Solution

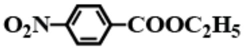
$E.N.$ of $Al = (1.5) \approx Be(1.5)$

#1330040

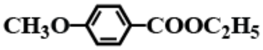
I



II



III



IV

The decreasing order of ease of alkaline hydrolysis for the following esters is:

A $IV > II > III > I$

[✓ B] $III > II > I > IV$

C $III > II > IV > I$

D $II > III > I > IV$

Solution

More is the electrophilic character of carbonyl group of ester faster is the alkaline hydrolysis.

#1330062

A process has $\Delta H = 200\, J\, mol^{-1}$ and $\Delta S = 40\, JK\, mol^{-1}$. Out of the values given above which the process will be sponteneous:

[✓ A] $5\, K$

B $4\, K$

C $20\, K$

D $12\, K$

Solution

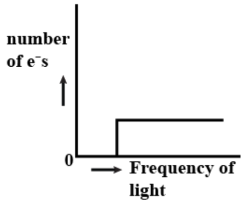
$\Delta G = \Delta H - T\Delta S$

$$T = \frac{\Delta H}{\Delta S} = \frac{200}{40} = 5K$$

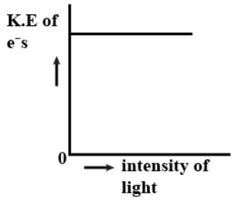
#1330082

Which of the graphs shown below does not represent the relationship between incident light and the electron ejected form metal surface?

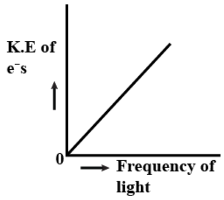
A



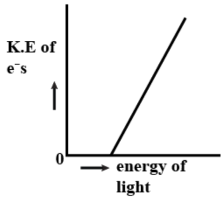
B



[✓ C]



D



Solution

$$E = W + \frac{1}{2}mv^2$$

$K.E. = hv - 4v_0$

$K.E. = hv + (-hv_0)$

$y = mx + \underline{C}$

#1330098

Which of the following is not and example of heterogeneous catalytic reaction?

A Ostwald's process

B Haber's process

[✓ **C**] Combustion of coal

D Hydrogenation of vegetable oils

Solution

Then is no catalyst is required for combustion of coal.

#1330119

The effect of lanthanoid contraction in the lanthanoid series of elements by and large means:

[✓ **A**] decrease in both atomic and ionic radii

B increase in atomic radii and decrease in ionic radii

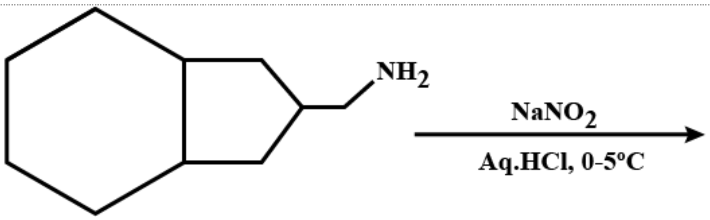
C increase in both atomic and ionic radii

D decrease in atomic radii and increase in ionic radii

Solution

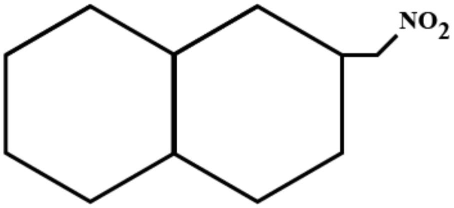
Due to lanthanoid contraction both atomic radii and ionic radii decrease decrease gradually in the lanthanoid series.

#1330137

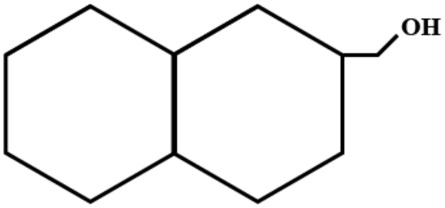


The major product formed in the reaction given below will be:

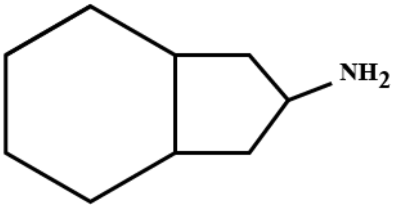
A



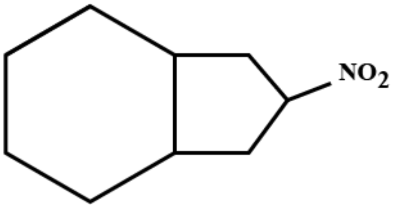
B



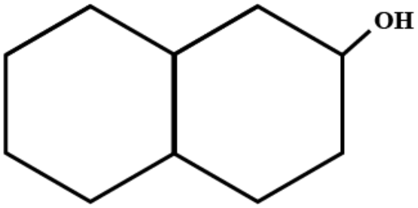
C



D

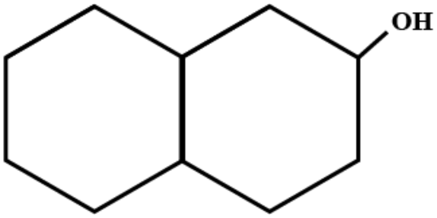


E



Solution

Answer should be

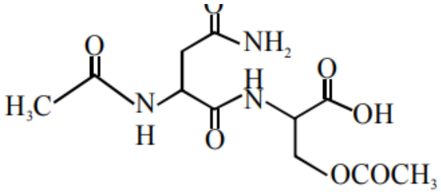


#1330196

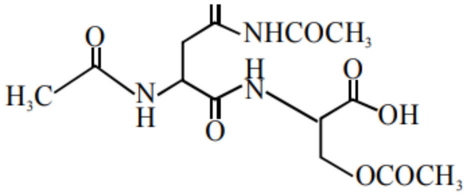
The correct structure of product 'P' in the following reaction is:

$$Asn - Ser + \underset{(excess)}{(CH_3CO_2)_2O} \xrightarrow{NEt_3} P$$

A



B



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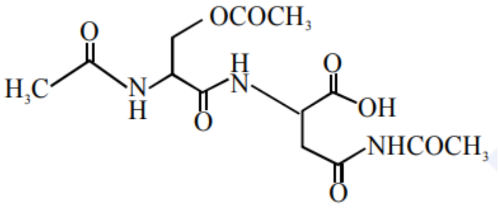
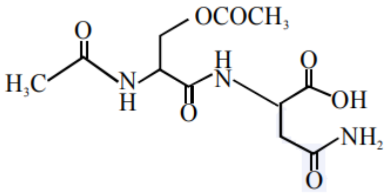
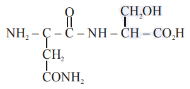


D



Solution

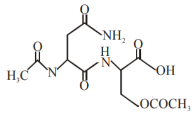
Asn-Ser is dipeptide having following structure



$NH_2-\underset{\underset{CONH_2}{|}}{\underset{CH_2}{|}}{C}-\overset{O}{\overset{||}{C}}-NH-\overset{CH_2OH}{\overset{|}{C}H}-CO_2H$

$Asn-Ser+\underset{excess}{(CH_3CO)_2O}\xrightarrow{NEt_3}P$

P is



#1330225

Which hydrogen is compound (E) is easily replaceable during bromination reaction in presence of light?

$$\underset{\delta}{CH_3}-\underset{\gamma}{CH_2}-\underset{\beta}{CH}=\underset{\alpha}{CH_2}$$

A $\beta-$ hydrogen

[✓ B] $\gamma-$ hydrogen

C $\delta-$ hydrogen

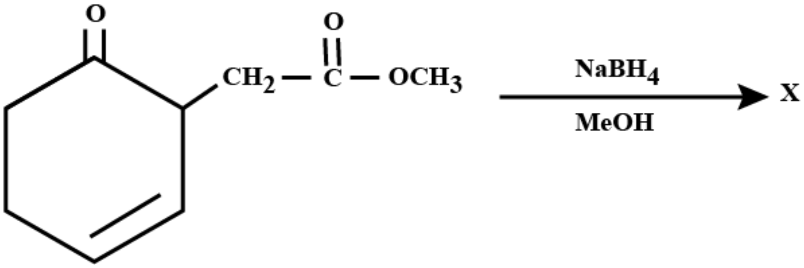
D $\alpha-$ hydrogen

Solution

$\gamma-Hydrogen$ is easily replacable during bromination reaction in presence of light , because Allylic substitution is being preferred.

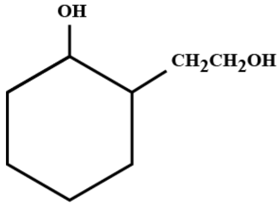
So, Option B is correct

#1330237

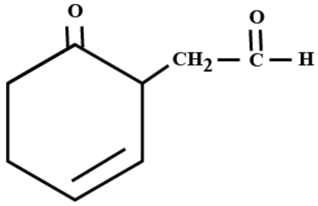


The major product 'X' formed in the following reaction is:

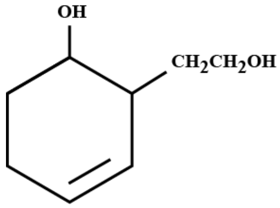
A



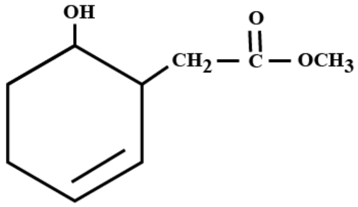
B



C



D



Solution

$NaBH_4$ cannot reduce the Ester Groups and double bonds, but can reduce keto-group to enol-group .So, The reactant reacts with $NaBH_4$ and $-OH$ is formed .Then in the 2nd Step , $-OMe$ is substituted and gives the product same as in the 1st step.

So, Option D is correct

#1330265

A mixture of $100\,m\,mol$ of $Ca(OH)_2$ and $2g$ of sodium sulphate was dissolved in water and the volume was made up to $100\,mL$. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are:

(Molar mass of $Ca(OH)_2, Na_2SO_4$ and $CaSO_4$ are $74, 143$ and $136\,g\,mol^{-1}$, respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

A $1.9\,g, 0.14\,mol\,L^{-1}$

B $13.6\,g, 0.14\,mol\,L^{-1}$

C $1.9\,g, 0.28\,mol\,L^{-1}$

D $13.6\,g, 0.28\,mol\,L^{-1}$

Solution

$Ca(OH_2) + Na_2SO_4 \rightarrow CaSO_4 + 2NaOH$

100 m mol	14 m mol	$----$	$----$
$----$	$----$	14 m mol	28 m mol

w = $14 \times 10^{-3} \times 136$ = $1.9\,g$

$[OH]^-$ = $\frac{28}{100}$ = 0.28

#1329451

Consider a triangular plot ABC with sides AB = 7cm, BC = 5cm and CA = 6cm. A vertical lamp-post at the mid point D of AC subtends an angle 30^o. The height (in m) of the lamp-post is?

A $7\sqrt{3}$

[✓ **B**] $\frac{2}{3}\sqrt{21}$

C $\frac{3}{2}\sqrt{21}$

D $2\sqrt{21}$

Solution

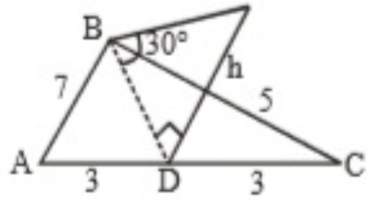
BD = $h\cot 30^o = h\sqrt{3}$

So, $7^2 + 5^2 = 2(h\sqrt{3})^2 + 3^2)$

$\Rightarrow 37 = 3h^2 + 9.$

$\Rightarrow 3h^2 = 28$

$\Rightarrow h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}.$



#1329468

Let f : R $\rightarrow$ R be a function such that $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3), x \in R$. Then $f(2)$ equal?

A 8

[✓ **B**] -2

C -4

D 30

Solution

$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2x f'(1) + f''(2)$ ·(1)

$\Rightarrow f''(x) = 6x + 2f'(1)$ ··(2)

$\Rightarrow f'''(x) = 6$ ·(3)

Put $x = 1$ in equation (1):

$f'(1) = 3 + 2f'(1) + f''(2)$ ··(4)

Put $x = 2$ in equation (2):

$f''(2) = 12 + 2f'(1)$ ·(5)

from equation (4) & (5):

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow 3f'(1) = -15$

$\Rightarrow f'(1) = -5$

$\Rightarrow f''(2) = 2$ ··(2)

put $x = 3$ in equation (3):

$f'''(3) = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$·

#1329476

If a circle C passing through the point $(4, 0)$ touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point $(1, -1)$, then the radius of C is?

A $\sqrt{57}$

B 4

C $2\sqrt{5}$

$\boxed{\text{D}}$ 5

Solution

$x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at $(1, -1)$

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through $(4, 0)$:

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow 20 + \lambda(5) = 0$

$\Rightarrow \lambda = -4$

$\therefore (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$

or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius $= \sqrt{16 + 25 - 16} = 5$.

#1329487

In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not option for any of the three courses is?

A 102

B 42

C 1

[✓ D] 38

Solution

Let $n(A) =$ number of students opted Mathematics $= 70,$

$n(B) =$ number of students opted Physics $= 46,$

$n(C) =$ number of students opted Chemistry $= 28,$

$n(A \cap B) = 23,$

$n(B \cap C) = 9,$

$n(A \cap C) = 14,$

$n(A \cap B \cap C) = 4,$

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

Si number of students not opted for any course $=$ Total $- n(A \cap B \cap C)$

$= 140 - 102 = 38.$

#1329496

The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is?

A 1365

B 1256

C 1465

[✓ D] 1356

Solution

$$\sum_{r=2}^{13} (7r + 2) = 7 \cdot \frac{2 + 13}{2} \times 6 + 2 \times 12$$

$= 7 \times 90 + 24 = 654$

$$\sum_{r=1}^{13} (7r + 5) = 7\left(\frac{1 + 13}{2}\right) \times 13 + 5 \times 13 = 702$$

Total $= 654 + 702 = 1356.$

#1329603

Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is?

A $\left(\frac{1}{2}, 4, -2\right)$

[✓ B] $\left(-\frac{1}{2}, 4, 0\right)$

C $(1, 3, 1)$

D $(1, 5, 1)$

Solution

$4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\Rightarrow 3-\lambda_2=2\lambda_1 \Rightarrow 2\lambda_1+\lambda_2=3 \cdot(1)$

Given $\vec{a}\cdot\vec{c}=0$

$\Rightarrow 6+6\lambda_1+3(\lambda_3-1)=0$

$\Rightarrow 2\lambda_1+\lambda_3=-1\,(2)$

Now $(\lambda_1,\lambda_2,\lambda_3)=(\lambda_1,3-2\lambda_1,\,-1-2\lambda_1)$

Now check the options, option (2) is correct.

#1329607

The equation of a tangent to the hyperbola $4x^2-5y^2=20$ parallel to the line $x-y=2$ is?

A $x-y+9=0$

B $x-y+7=0$

[✓ C] $x-y+1=0$

D $x-y-3=0$

Solution

Hyperbola $\dfrac{x^2}{5}-\dfrac{y^2}{4}=1$

Slope of tangent $=1$

Equation of tangent $y=x\pm\sqrt{5-4}$

$\Rightarrow y=x\pm1$

$\Rightarrow y=x+1$ or $y=x-1$.

#1329615

If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, $(k>0)$, is 1 square unit. Then k is?

[✓ A] $\dfrac{1}{\sqrt{3}}$

B $\dfrac{2}{\sqrt{3}}$

C $\dfrac{\sqrt{3}}{2}$

D $\sqrt{3}$

Solution

Area bounded by $y^2=4ax$ & $x^2=4by$, a, b $\neq 0$ is $\left|\dfrac{16ab}{3}\right|$

by using formula: $4a=\dfrac{1}{k}=4b,\ k>0$

$$\text{Area }=\left|\frac{16\cdot\frac{1}{4k}\cdot\frac{1}{4k}}{3}\right|=1$$

$\Rightarrow k^2=\dfrac{1}{3}$

$\Rightarrow k=\dfrac{1}{\sqrt{3}}$.

#1329631

Let $f(x) = \begin{cases} max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then S?

A Is an empty set

B Equals $\{-2, -1, 1, 2\}$

$\boxed{\textbf{C}}$ Equals $\{-2, -1, 0, 1, 2\}$

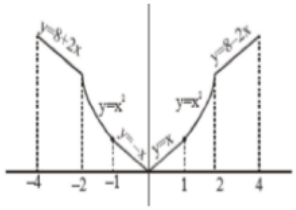
D Equals $\{-2, 2\}$

Solution

$$f(x) = \begin{cases} 8 + 2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8 - 2x, & 2 < x \le 4 \end{cases}$$

$f(x)$ is not differentiable at $x = \{-2, -1, 0, 1, 2\}$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}.$



#1329642

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c).

A $(1, 1, 0)$

B $\left(\frac{1}{2}, 2, 3\right)$

C $\left(\frac{1}{2}, 2, 0\right)$

$\boxed{\textbf{D}}$ $(1, 1, 3)$

Solution

Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b)m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b)m = (b - 2a)m^3$

We get all options are correct for $m = 0$

(common normal x-axis)

Ans. (1), (2), (3), (4)

Remark:

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered traid (a, b, c)?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$m^2 = \frac{c}{2a - b} - 2 > 0 \Rightarrow \frac{c}{2a - b} > 2$

Now according to options, option 4 is correct.

#1329652

The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2\theta + \cos^4 2\theta = \frac{3}{4}$ is?

[✓ **A**] $\frac{\pi}{2}$

B π

C $\frac{3\pi}{8}$

D $\frac{5\pi}{4}$

Solution

$\sin^2\theta + \cos^4 2\theta = \frac{3}{4}, \theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} = \cos^2\frac{\pi}{4}$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}, n \in I$

$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{\pi}{2} - \frac{\pi}{8}$

Sum of solutions $\frac{\pi}{2}$.

#1329664

Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then?

[✓ **A**] $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$

B $Re(z) = 0$

C $|z| = \sqrt{\frac{5}{2}}$

D $Im(z) = 0$

Solution

Bonus.

$3|z_1| = 4|z_2|$

$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$

$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$

Let $\frac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a}$

$= \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$

Now all options are incorrect

Remark

There is a misprint in the problem actual problem should be:

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| = 2|z_2|$.

If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\frac{3z_1}{2z_2}\right| = 1$

Let $\frac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$

$= a + \frac{1}{a} = 2\cos\theta$

$\therefore Im(z) = 0$

Now option (4) is correct.

#1329673

If the system of equations $x + y + z = 5$, $x + 2y + 3z = 9$, $x + 3y + \alpha z = \beta$ has infinitely many solutions, then $\beta - \alpha$ equals?

A 5

B 18

C 21

$\boxed{\text{D}}$ 8

Solution

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha - 1 \end{vmatrix} = (\alpha - 1) - 4 = (\alpha - 5)$$

for infinite solutions $D = 0 \Rightarrow \alpha = 5$

$$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta - 15 & -2 & 5 \end{vmatrix} = 0$$

$\Rightarrow 2 + \beta - 15 = 0$

$\Rightarrow \beta - 13 = 0$

on $\beta = 13$ we get $D_y = D_z = 0$

$\alpha = 5, \beta = 13$.

#1329679

The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$ is?

$\boxed{\text{A}}$ $\frac{\sqrt{5}}{2}$

B $\frac{5}{4}$

C $\frac{3}{2}$

D $\frac{\sqrt{3}}{2}$

Solution

Let points $\left(\frac{3}{2}, 0\right), (t^2, t) t > 0$

Distance $= \sqrt{t^2 + \left(t^2 - \frac{3}{2}\right)^2}$

$= \sqrt{t^2 - 2t^2 + \frac{9}{4}} = \sqrt{(t^2 - 1)^2 + \frac{5}{4}}$

So minimum distance is $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$.

#1329688

Consider the quadratic equation $(c - 5)x^2 - 2cx + (c - 4) = 0, c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval $(0, 2)$ and its other root lies in the interval $(2, 3)$. Then the number of elements in S is?

$\boxed{\text{A}}$ 11

B 18

C 10

D 12

Solution

Let $f(x) = (c-5)c^2 - 2cx + c - 4$

$\therefore f(0)f(2) < 0 \cdot (1)$

& $f(2)f(3) < 0$ (2)

from (1) & (2)

$(c-4)(c-24) < 0$

& $(c-24)(4c-49) < 0$

$\Rightarrow \frac{49}{4} < c < 24$

$\therefore s = \{13, 14, 15, \ldots..23\}$

Number of elements in set S = 11.

#1329702

$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right) = \frac{k}{21}$, then k equals?

A 200

B 50

$\boxed{\text{C}}$ 100

D 400

Solution

$$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^2 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^2}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$$

$\Rightarrow 100 = k.$

#1329716

Let $d \in R$, and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0, 2\pi]$. If the minimum value of $det(A)$ is 8, then a value of d is?

A -7

B $2(\sqrt{2}+2)$

$\boxed{\text{C}}$ -5

D $2(\sqrt{2}+1)$

Solution

$$detA = \begin{vmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 5 & 2\sin\theta - d & -\sin\theta + 2 + 2d \end{vmatrix}$$

$(R_1 \rightarrow R_1 + R_3 - 2R_2)$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta + 2 & d \\ 5 & 2\sin\theta - d & 2 + 2d - \sin\theta \end{vmatrix}$$

$= (2 + \sin\theta)(2 + 2d - \sin\theta) - d(2\sin\theta - d)$

$= 4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$= d^2 + 4d + 4 - \sin^2\theta$

$= (d + 2)^2 - \sin^2\theta$

For a given d, minimum value of $det(A) = (d + 2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5.

#1329720

If the third term in the binomial expansion of $(1 + x^{log_2 x})^5$ equals 2560, then a possible value of x is?

A $2\sqrt{2}$

B $\frac{1}{8}$

C $4\sqrt{2}$

$\boxed{\textbf{D}}$ $\frac{1}{4}$

Solution

$(1 + x^{log_2 x})^5$

$T_3 = {}^5C_2 \cdot (x^{log_2 x})^2 = 2560$

$\Rightarrow 10 \cdot x^{2 log_2 x} = 2560$

$\Rightarrow x^{2 log_2 x} = 256$

$\Rightarrow 2(log_2 x)^2 = log_2 256$

$\Rightarrow 2(log_2 x)^2 = 8$

$\Rightarrow (log_2 x)^2 = 4$

$\Rightarrow log_2 x = 2$ or -2

$x = 4$ or $\frac{1}{4}$.

#1329726

If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is?

A (3, 4)

$\boxed{\textbf{B}}$ (2, 2)

C (4, 4)

D (4, 3)

Solution

Line intersects the x-axis at $x = 8,\ y = 0$

$\Rightarrow A = (8, 0)$

Line intersects the y-axis at $x = 0,\ y = 6$

$\Rightarrow B = (0, 6)$

Incenter of triangle OAB $= \left(\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}\right), \left(\dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

Where $O = \left(x_1, y_1\right) = (0, 0),\ A = \left(x_2, y_2\right) = (8, 0),\ B = \left(x_3, y_3\right) = (0, 6)$

$\Rightarrow AB = 10,\ OB = 6,\ OA = 8$

$\Rightarrow$ Incenter $= \left(\dfrac{10.0 + 6.8 + 8.0}{10 + 8 + 6}, \dfrac{10.0 + 6.0 + 8.6}{10 + 8 + 6}\right)$

$\Rightarrow$ incenter $= (2, 2)$

#1329732

The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are $1, 3$ and 8, then a ratio of other two observations is?

A $4:9$

B $6:7$

C $5:8$

D $10:3$

Solution

Let two observations are x_1 & x_2

mean $= \dfrac{\Sigma x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13 \cdot (1)$

variance $(\sigma^2) = \dfrac{\Sigma x_i^2}{5} - 25 = 9.20$

$\Rightarrow \Sigma x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97 \cdot\cdot (2)$

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9.$

#1329741

A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of ΔPQR is a line?

A Parallel to x-axis

B With slope $\dfrac{2}{3}$

C With slope $\dfrac{3}{2}$

D Parallel to y-axis

Solution

Let the centroid of △PQR is (h, k) & P is (α, β), then

$\frac{\alpha + 1 + 3}{3} = h$ and $\frac{\beta + 4 - 2}{3} = k$

$\alpha = (3h - 4) \quad \beta = (3k - 4)$

Point $P(\alpha, \beta)$ lies on line $2x - 3y + 4 = 0$

$\therefore 2(3h - 4) - 3(3k - 2) + 4 = 0$

$\Rightarrow$ locus is $6x - 9y + 2 = 0$.

#1329756

If $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}, x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals?

$\boxed{\text{A}}$ $\frac{1}{3} + e^6$

B $\frac{1}{3}$

C $-\frac{4}{3}$

D $\frac{1}{3} + e^3$

Solution

$\frac{dy}{dx} + 3\sec^2 x \cdot y = \sec^2 x$

I.F. $= e^{3\int \sec^2 x dx} = e^{3\tan x}$

or $y \cdot e^{3\tan x} = \int \sec^2 x \cdot e^{2\tan x} dx$

or $y \cdot e^{3\tan x} = \frac{1}{3} e^{3\tan x} + C \cdot (1)$

Given

$y\left(\frac{\pi}{4}\right) = \frac{4}{3}$

$\therefore \frac{4}{3} \cdot e^3 = \frac{1}{3} e^3 + C$

$\therefore C = e^3$

Now put $x = -\frac{\pi}{4}$ in equation (1)

$\therefore y \cdot e^{-3} = \frac{1}{3} e^{-3} + e^3$

$\therefore y = \frac{1}{3} + e^6$

$\therefore y\left(-\frac{\pi}{4}\right) = \frac{1}{3} + e^6.$

#1329765

The plane passing through the point $(4, -1, 2)$ and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point.

A $(-1, -1, -1)$

B $(-1, -1, 1)$

C $(1, 1, -1)$

$\boxed{\text{D}}$ $(1, 1, 1)$

Solution

Let $\vec{n}$ be the normal vector to the plane passing through $(4, -1, 2)$ and parallel to the lines L_1 & L_2

then $\vec{n} = \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \hline 3 & -1 & 2 \\ 1 & 2 & 3 \end{array}$

$\therefore \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

$-1(x-4) - 1(y+1) + 1(z-2) = 0$

$\therefore x + y - z - 1 = 0$

Now check options.

#1329776

Let $I = \int_a^b (x^4 - 2x^2)dx$. If I is minimum then the ordered pair (a, b) is?

A $(-\sqrt{2}, 0)$

[✓ B] $(-\sqrt{2}, \sqrt{2})$

C $(0, \sqrt{2})$

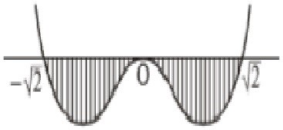
D $(\sqrt{2}, -\sqrt{2})$

Solution

Let $f(x) = x^2(x^2 - 2)$

As long as $f(x)$ lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I.



#1329787

If $5, 5r, 5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to?

A $\frac{3}{2}$

B $\frac{3}{4}$

C $\frac{5}{4}$

[✓ D] $\frac{7}{4}$

Solution

$5, 5r, 5r^2$ sides of triangle,

$5 + 5r > 5r^2$ ...(1)

$5 + 5r^2 > 5r$ ...(2)

$5r + 5r^2 > 5$ ...(3)

From (1) $r^2 - r - 1 < 0$,

$$\left[r - \left(\frac{1+\sqrt{5}}{2}\right)\right]\left[r - \left(\frac{1-\sqrt{5}}{2}\right)\right] < 0$$

$$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right) \quad ...(4)$$

from (2),

$r^2 - r + 1 > 0 \Rightarrow r \in R$ ...(5)

from (3),

$r^2 + r - 1 > 0$

So, $\left(r + \frac{1+\sqrt{5}}{2}\right)\left(r + \frac{1-\sqrt{5}}{2}\right) > 0$

$$r \in \left(-\infty, -\frac{1+\sqrt{5}}{2}\right) \cup \left(-\frac{1-\sqrt{5}}{2}, \infty\right) \quad ...(6)$$

from (4), (5), (6),

$$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$$

#1329794

Consider the statement: " $P(n) : n^2 - n + 41$ is prime". Then which one of the following is true?

A P(5) is false but P(3) is true

B Both P(3) and P(5) are false

C P(3) is false but P(5) is true

$\boxed{\text{D}}$ Both P(3) and P(5) are true

Solution

$P(n) : n^2 - n + 41$ is prime

$P(5) = 61$ which is prime

$P(3) = 47$ which is also prime.

#1329805

Let A be a point on the line $\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$ and $B(3, 2, 6)$ be a point in the space. Then the value of μ for which the vector $\overline{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is?

A $\frac{1}{2}$

B $-\frac{1}{4}$

$\boxed{\text{C}}$ $\frac{1}{4}$

D $\frac{1}{8}$

Solution

Let point A is

$(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$

and point B is $(3, 2, 6)$

then $\bar{AB} = (2+3\mu)\hat{i}+(3-\mu)\hat{j}+(4-5\mu)\hat{k}$

which is parallel to the plane $x-4y+3z=1$

$\therefore 2+3\mu-12+4\mu+12-15\mu=0$

$8\mu=2$

$\mu=\frac{1}{4}.$

#1329811

For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then, $\lim_{x\to 1+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$.

A Equals -1

B Equals 1

C Does not exist

[✓ D] Equals 0

Solution

$$\lim_{x\to 1^+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$= \lim_{x\to 1^+} \frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$$

$$= \lim_{x\to 1^+}\left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0.$$

#1329829

An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered $1, 2, 3, \ldots, 9$ is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is?

A $\frac{13}{36}$

B $\frac{19}{36}$

[✓ C] $\frac{19}{72}$

D $\frac{15}{72}$

Solution

$P(7\ or\ 8) = P(H)P(7\ or\ 8) + P(T)P(7\ or\ 8) = \frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{9}=\frac{19}{72}.$

#1329859

Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$. Then $\int \frac{(\sin^{\pi}\theta-\sin\theta)^{\frac{1}{\pi}}\cos\theta}{\sin^{\pi+1}\theta}d\theta$ is equal to: (Where C is a constant of integration)

A $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{\pi+1}\theta}\right)^{\frac{\pi+1}{\pi}}+C$

B $\frac{1}{n^2+1}\left(1-\frac{1}{\sin^{\pi-1}\theta}\right)^{\frac{\pi+1}{\pi}}+C$

$\boxed{C}$ $\frac{1}{n-1}\left(1-\frac{1}{\sin^{\pi-1}\theta}\right)^{\frac{\pi+1}{\pi}}+C$

D $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{\pi-1}\theta}\right)^{\frac{\pi+1}{\pi}}+C$

Solution

$$\int\frac{(\sin^{\pi}\theta-\sin\theta)^{1/\pi}\cos\theta}{\sin^{\pi+1}\theta}d\theta$$

$$=\int\frac{\sin\theta\left(1-\frac{1}{\sin^{\pi-1}\theta}\right)^{1/\pi}d\theta}{\sin^{\pi+1}\theta}$$

Put $1-\frac{1}{\sin^{\pi-1}\theta}=t$

So $\frac{(n-1)}{\sin^{\pi}\theta}\cos\theta d\theta=dt$

Now $\frac{1}{n-1}\int(t)^{1/\pi}dt$

$$=\frac{1}{(n-1)}\frac{(t)^{\frac{1}{\pi}+1}}{\frac{1}{n}+1}+C$$

$$=\frac{1}{(n-1)}\left(1-\frac{1}{\sin^{\pi-1}\theta}\right)^{\frac{1}{\pi}+1}+C$$