

#161182

Topic: Application of Binomial Expansion

The sum of coefficient of even powers of  $x$  in  $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$  is

A 23

**B** 24

C 18

D 21

Solution

 $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6 = ({}^6C_0 \cdot x^6 + {}^6C_2 \cdot x^4(x^3 - 1) + {}^6C_4 x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3)$  Terms with even powers of

 $x = 2({}^6C_0 \cdot x^6 - {}^6C_2 \cdot x^4 + {}^6C_4 x^2 + {}^6C_6(-1 - 3x^6))$  Coefficients  $= 2({}^6C_0 - {}^6C_2 + {}^6C_4 + {}^6C_6 - 3 \cdot {}^6C_6) = 2(15 - 3) = 24$ 

#161190

Topic: Trigonometric Functions

Let  $\sin(\alpha - \beta) = \frac{5}{13}$  and  $\cos(\alpha + \beta) = \frac{3}{5}$ , then  $\tan(2\alpha)$  is equal to (Here  $\alpha, \beta \in \left(0, \frac{\pi}{4}\right)$ )**A**  $\frac{63}{16}$ B  $\frac{61}{16}$ C  $\frac{65}{16}$ D  $\frac{32}{9}$ 

Solution

 $0 < \alpha < \frac{\pi}{4}, 0 < \beta < \frac{\pi}{4} \Rightarrow 0 < \alpha + \beta < \frac{\pi}{2}$  and  $-\frac{\pi}{4} < \alpha - \beta < \frac{\pi}{4}$  Now  $\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \cos(\alpha - \beta) = \frac{12}{13}$  and  $\cos(\alpha + \beta) = \frac{3}{5} \Rightarrow \sin(\alpha + \beta) = \frac{4}{5}$  Now

$$\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)] = -\frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} = \frac{63}{16}$$

#161192

Topic: Position of a Point/Line w.r.t Circle

The line  $x + y = n, n \in \mathbb{N}$  makes intercepts with  $x^2 + y^2 = 16$ . Then the sum of square of all possible intercepts.A  $\frac{105}{4}$ 

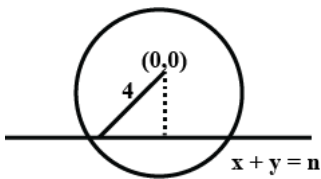
B 105

**C** 210D  $\frac{105}{2}$

**Solution**

$$P = \frac{n}{\sqrt{2}} \text{ to make the intercept } \frac{n}{\sqrt{2}} < 4 \Rightarrow n < 4\sqrt{2} \text{ Length of intercepts } = 2\sqrt{r^2 - p^2} = 2\sqrt{16 - n^2/2} \text{ Square of intercept } = 4 \times \left(16 - \frac{n^2}{2}\right), n \in \mathbb{N} \text{ Sum of squares of intercept}$$

$$= 4 \times \left( \left(16 - \frac{1}{2}\right) + \left(16 - \frac{4}{2}\right) + \left(16 - \frac{16}{2}\right) + \left(16 - \frac{25}{2}\right) \right) = 2 \left( 80 - \frac{1}{2} \times 22 \right) = 210$$



#161194

Topic: Special Integrals (Trigonometric Functions)

$$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx \text{ is equal to}$$

- A**  $x + 2\sin x + \sin 2x + C$
- B**  $x + 2\cos x + \sin 2x + C$
- C**  $x + 2\sin x + \sin 2x + C$
- D**  $x + 2\sin x - \sin 2x + C$

**Solution**

$$\frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx = \frac{2\sin \frac{5x}{2} \cos \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} dx = \int \left( \frac{\sin 3x + \sin 2x}{\sin x} \right) dx = \int 2\cos x dx + \int (3 - 4\sin^2 x) dx = 2\sin x + x + \sin 2x + C$$

#161195

Topic: Area of Bounded Regions

The area bounded by the curve  $y \leq x^2 + 3x$ ,  $0 \leq y \leq 4$ ,  $0 \leq x \leq 3$ , is

- A**  $\frac{59}{6}$
- B**  $\frac{57}{4}$
- C**  $\frac{59}{3}$
- D**  $\frac{57}{6}$

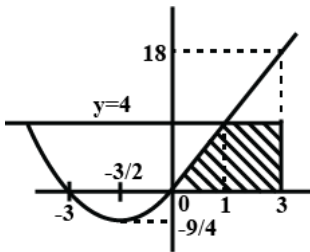
**Solution**

$$y = x^2 + 3x = 4 \Rightarrow x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

$$x = 1$$

$$\text{area} = \int_0^1 (x^2 + 3x) dx + 2(4) = \frac{1}{3} + \frac{3}{2} + 8 = \frac{11}{6} + 8 = \frac{59}{6}$$



#1611207

Topic: Truth Tables

"If you are born in India then you are citizen of India" Contrapositive of this statement is

- A If you are born in India then you are not citizen of India
- ☒ B If you are not citizen of India then you are not born in india
- C If you are citizen of India then you are not born in India
- D If you are citizen of India then you are born in India

**Solution**

Contrapositive of  $p \Rightarrow q$  is  $\sim q \Rightarrow \sim p$  Hence answer is "If you are not citizen of India then you are not born in India."

#1611210

Topic: Operations on Matrices

Let  $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$  and  $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  then  $\alpha$  may be

- A 0
- B  $\frac{\pi}{32}$
- ☒ C  $\frac{\pi}{64}$
- D  $\frac{\pi}{16}$

**Solution**

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$\text{similarly we observe that } A^n = \begin{bmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{bmatrix}$$

hence  $\cos 32\alpha = 0$  and  $\sin 32\alpha = 1$

$$32\alpha = 2n\pi + \frac{\pi}{2}$$

$$\alpha = \frac{n\pi}{16} + \frac{\pi}{64}, n \in \mathbb{Z}$$

#1611214

Topic: Tangent

Shortest distance between the curve  $y^2 = x - 2$  and  $y = x$  is

- A greater than 4
- B

 less than 2
- C greater than 3
- D greater than 2

**Solution**

Shortest distance between  $y^2 = x - 2$  and  $y = x$

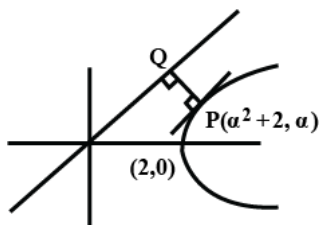
$\frac{dy}{dx}$  at point  $P$  will be 1.

Differentiating the curve

$$2yy' = 1 \Rightarrow y' = \frac{1}{2y} = \frac{1}{2a} = 1 \quad \therefore P\left(\frac{9}{4}, \frac{1}{2}\right)$$

$\therefore$  minimum distance =  $PQ$

$$= \left| \frac{\frac{9}{4} - \frac{1}{2}}{\sqrt{2}} \right| = \frac{7}{4\sqrt{2}}$$



#1611220

Topic: L'Hospital's Rule

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}} =$$

A  $\sqrt{2}$

B 2

C 4

☒ D  $4\sqrt{2}$

Solution

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}} &= \lim_{x \rightarrow 0} \frac{\sin^2 x (\sqrt{2} + \sqrt{1 + \cos x})}{2 - 1 - \cos x} \\ &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \left( \frac{1 - \cos x}{x^2} \right) (\sqrt{2} + \sqrt{1 + \cos x}) = \frac{2\sqrt{2}}{1/2} = 4\sqrt{2} \end{aligned}$$

#1611223

Topic: Permutation Involving Restrictions

How many 9 digit number can be formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 such that odd numbers occur at even places.

A 160

B 175

☒ C 180

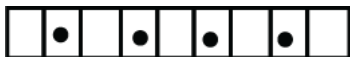
D 220

**Solution**

1, 1, 2, 2, 2, 2, 3, 4, 4

odd numbers occur at even places

$${}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{4!2!} = 4 \times 3 \times 15 = 180$$

**#1611226**

**Topic:** Properties of Definite Integral

Let  $g(x) = \ln x$  and  $f(x) = \left( \frac{1 - x \cos x}{1 + x \cos x} \right)$  then  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} g(f(x)) dx$  is equal to

- ☒ A  $\ln 1$
- ☐ B  $\ln 2$
- ☐ C  $\ln e$
- ☐ D  $\ln 4$

**Solution**

$$g(x) = \ln(x), f(x) = \frac{1 - x \cos x}{1 + x \cos x} \text{ and } g(f(x)) = \ln\left(\frac{1 - x \cos x}{1 + x \cos x}\right)$$

$$I = \int_{-\pi/4}^{\pi/4} \ln\left(\frac{1 - x \cos x}{1 + x \cos x}\right) dx$$

$$x \rightarrow a + b - x$$

$$x \rightarrow -x$$

$$I = \int_{-\pi/4}^{\pi/4} \ln\left(\frac{1 + x \cos x}{1 - x \cos x}\right) dx$$

$$\text{Adding, } 2I = \int_{-\pi/4}^{\pi/4} \ln(1) dx = 0 \Rightarrow I = 0$$

**#1611228**

**Topic:** Binomial Coefficients

Let  $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20}$ . then sum of this series is

- ☐ A  $16 \cdot 2^{22}$
- ☐ B  $8 \cdot 2^{20}$
- ☐ C  $8 \cdot 2^{21}$
- ☒ D  $16 \cdot 2^{21}$

**Solution**

$$\begin{aligned}
 2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20} &= \sum_{r=0}^{20} (3r+2) \cdot {}^{20}C_r \\
 &= 3 \sum_{r=0}^{20} r \cdot {}^{20}C_r + 2 \sum_{r=0}^{20} {}^{20}C_r = 3 \times 20 \sum_{r=1}^{20} {}^{19}C_{r+1} + 2 \cdot (2^{20}) = 60 \cdot 2^{19} + 2 \cdot 2^{20} = 16 \cdot 2^{21}
 \end{aligned}$$

#1611229

Topic: Special Series

Sum of the natural number between 100 and 200 whose HCF with 91 should be more than 1

- A 1121
- B 3210
- ☒ C 3121
- D 1520

**Solution**

Natural numbers between 100 &amp; 200.

101, 102, ..., 199.

Either divide by 7 or divide by 13.

(sum of numbers (divide by 7) + (sum of number divided by 13) - (sum of number of divide by 91))

$$\begin{aligned}
 \sum_{r=1}^{14} (98 + 7r) + \sum_{r=1}^8 (91 + 13r) - (182) &= \left( 98 \times 14 + 7 \cdot \frac{14 \times 15}{2} \right) + \left( (91 \times 8) + 13 \times \frac{8 \times 9}{2} \right) - 182 \\
 &= 1372 + 735 + 728 + 468 - 182 = 3121
 \end{aligned}$$

#1611230

Topic: Variance and Standard Deviation

If mean and variance of 7 variates are 8 and 16 respectively and five of them are 2, 4, 10, 12, 14 then find the product of remaining two variates

- A 49
- ☒ B 48
- C 45
- D 40

**Solution**

Let remaining two variates are a and b then

$$\frac{a + b + 2 + 4 + 10 + 12 + 14}{7} = 8$$

$$\text{and } \frac{a^2 + b^2 + 4 + 16 + 100 + 144 + 196}{7} - (8)^2 = 16$$

$$\Rightarrow a + b = 14 \text{ and } a^2 + b^2 = 100$$

$$\Rightarrow ab = \frac{(a + b)^2 - (a^2 + b^2)}{2} = \frac{196 - 100}{2} = 48$$

#1611232

Topic: Maths

If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 2x + 2 = 0$  then find minimum value of n such that  $\left(\frac{\alpha}{\beta}\right)^n = 1$

- ☒ A 4
- ☐ B 3
- ☐ C 2
- ☐ D 5

#1611240

Topic: Linear Differential Equation

Solution of differential equation  $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$  is

- ☐ A  $y = \frac{\tan^{-1}x}{x^2 + 1} + C$
- ☐ B  $y = \tan^{-1}x + C$
- ☒ C  $y(x^2 + 1) = \tan^{-1}x + C$
- ☐ D  $y(\tan^{-1}x) = x^2 + C$

**Solution**



$$(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$$

$$\frac{dy}{dx} + \frac{2x}{x^2 + 1}y = \frac{1}{(x^2 + 1)^2}$$

Linear D.E.

$$I.F. = e^{\int \frac{2x}{x^2 + 1} dx} = e^{\ln(x^2 + 1)} = x^2 + 1$$

$$\therefore y(I.F.) = \int Q(I.F.) dx$$

$$y(x^2 + 1) = \int \frac{1}{(x^2 + 1)^2} (x^2 + 1) dx$$

$$y(x^2 + 1) = \tan^{-1}x + C$$

#1611256

Topic: Algebra of Real Functions

If  $f(x) = \log_e \left( \frac{1-x}{1+x} \right)$  then  $f\left(\frac{2x}{1+x^2}\right)$  is equal to

A  $f(x)$

☒ B  $2f(x)$

C  $-2f(x)$

D  $(f(x))^2$

Solution

$$f(x) = \ln \left( \frac{1-x}{1+x} \right)$$

$$f\left(\frac{2x}{1+x^2}\right) = \ln \left( \frac{1 - \frac{2x}{1+x^2}}{1 + \frac{2x}{1+x^2}} \right) = \ln \left( \frac{1+x^2-2x}{1+x^2+2x} \right) = \ln \left( \left( \frac{1-x}{1+x} \right)^2 \right) = 2 \ln \left( \frac{1-x}{1+x} \right) = 2f(x)$$

#1611261

Topic: Multiplication Theorem

Given that  $A \subset B$ , then identify the correct statement

A  $P(A/B) = P(A)$

B  $P(A/B) \leq P(A)$

☒ C  $P(A/B) \geq P(A)$

D  $P(A/B) = P(A) - P(B)$

Solution

$$A \subset B$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \neq P(A) \text{ (always)}$$

$$P(A/B) = \frac{P(A)}{P(B)} \geq P(A)$$

#1611266

Topic: Application of Matrices and Determinants

Find the value of  $c$  for which the following equations have non trivial solutions :

$$cx - y - z = 0$$

$$-cx + y - cz = 0$$

$$x + y - cz = 0$$

A  $\frac{1}{2}$

**B**  $-1$

C  $2$

D  $0$

Solution

$$\begin{vmatrix} c & -1 & -1 \\ -c & 1 & -c \\ 1 & 1 & -c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & -(1+c) \\ -c & 1 & -c \\ 1 & 1 & -c \end{vmatrix} = 0$$

$$\Rightarrow -(1+c)(-c-1) = 0$$

$$\Rightarrow c = -1$$

#1611277

Topic: Inverse Trigonometric Functions

Let  $2y = \left( \cot^{-1} \left( \frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x} \right) \right)^2$  then  $\frac{dy}{dx}$  is equal to

**A**  $x - \frac{\pi}{6}$

B  $x + \frac{\pi}{6}$

C  $2x - \frac{\pi}{6}$

D  $2x - \frac{\pi}{3}$

**Solution**

$$2y = \left( \cot^{-1} \left( \frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x} \right) \right)^2$$

$$2y = \left( \cot^{-1} \left( \frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} \right) \right)^2$$

$$2y = \left( \cot^{-1} \left( \tan \left( \frac{\pi}{3} + x \right) \right) \right)^2$$

$$2y = \left( \frac{\pi}{2} - \tan^{-1} \left( \tan \left( \frac{\pi}{3} + x \right) \right) \right)^2 = \left( \frac{\pi}{2} - \left( \frac{\pi}{3} + x \right) \right)^2$$

$$2y = \left( x - \frac{\pi}{6} \right)^2$$

$$2y = x^2 - \frac{\pi}{3}x + \frac{\pi^2}{36}$$

$$y' = x - \frac{\pi}{6}$$

**#1611280**

**Topic:** Maxima and Minima

Let  $S_1$  is set of minima and  $S_2$  is set of maxima for the curve  $y = 9x^4 + 12x^3 - 36x^2 - 25$ , then

**A**  $S_1 = \{-2, -1\}; S_2 = \{0\}$

**B**  $S_1 = \{-2, 1\}; S_2 = \{0\}$

**C**  $S_1 = \{-2, 1\}; S_2 = \{-1\}$

**D**  $S_1 = \{-2, 2\}; S_2 = \{0\}$

**Solution**

$$y = 9x^4 + 12x^3 - 36x^2 - 25$$

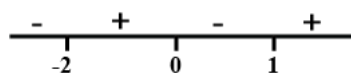
$$\frac{dy}{dx} = 36x^3 + 36x^2 - 72x$$

$$= 36x(x^2 + x - 2)$$

$$= 36x(x+2)(x-1)$$

$\{-2, 1\} \rightarrow$  points of minima

$\{0\} \rightarrow$  point of maxima



#1611285

Topic: Nature of the Function

Let  $f''(x) > 0$  and  $\phi(x) = f(x) + f(2-x)$ ,  $x \in (0, 2)$  be a function, then the function  $\phi(x)$  is

A increasing in  $(0, 1)$  and decreasing  $(1, 2)$

☒ B decreasing in  $(0, 1)$  and increasing  $(1, 2)$

C increasing in  $(0, 2)$

D decreasing in  $(0, 2)$

Solution

$$f''(x) > 0, y = f(x); x \in (0, 2)$$

$$\phi(x) = f(x) + f(2-x)$$

$$\phi'(x) = f'(x) - f'(2-x)$$

for  $\phi(x)$  to be increasing

$$\phi'(x) > 0 \Rightarrow f'(x) > f'(2-x)$$

$$\Rightarrow x > 2-x \quad (f'(x) \text{ is increasing in } (0, 2))$$

$$\Rightarrow x > 1$$

$$\Rightarrow x \in (1, 2)$$

For  $\phi(x)$  to be decreasing

$$\phi'(x) < 0 \Rightarrow f'(x) < f'(2-x)$$

$$\therefore x \in (0, 1)$$

#1611291

Topic: Properties of Triangles

Let vertices of the triangle  $ABC$  is  $A(0, 0)$ ,  $B(0, 1)$  and  $C(x, y)$  and perimeter is 4 then the locus of C is :

A  $9x^2 + 8y^2 + 8y = 16$

B  $8x^2 + 9y^2 + 9y = 16$

☒ C  $9x^2 + 8y^2 - 8y = 16$

D  $8x^2 + 9y^2 - 9x = 16$

**Solution**

$$\sqrt{x^2 + y^2} + \sqrt{x^2 + (y-1)^2} = 3$$

$$\Rightarrow x^2 + y^2 - 2y + 1 = 9 + x^2 + y^2 - 6\sqrt{x^2 + y^2}$$

$$3\sqrt{x^2 + y^2} = 4 + y$$

$$9x^2 + 9y^2 = 16 + y^2 + 8y$$

#1611297

Topic: Position of Points Relative to a Line

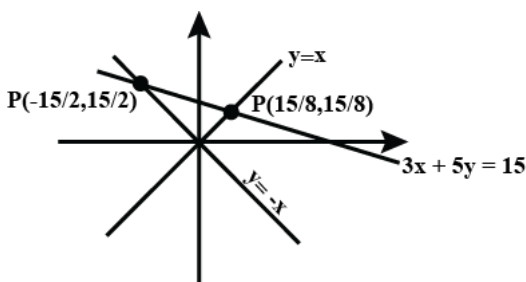
Let the equation of a line is  $3x + 5y = 15$  and a point P on this line is equidistant from x and y axis. In which quadrant the point P lies ?

- A**  $1^{st}$
- B**  $3^{rd}$
- C**  $4^{th}$
- D** None of these

**Solution**

From the figure,

Intersection of the original line with  $y = x$  and  $y = -x$  will give desired points P in  $1^{st}$  and  $2^{nd}$  quadrant.



#1611333

Topic: Lines

The perpendicular distance of point  $(2, -1, 4)$  from the line  $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$  lies between

- A** (2, 3)
- B** (3, 4)
- C** (4, 5)
- D** (1, 2)

**Solution**

Let the foot of perpendicular from  $P(2, -1, 4)$  to the given line be  $A(10\lambda - 3, -7\lambda + 2, \lambda)$ .  $\vec{PA} \cdot (10\hat{i} - 7\hat{j} + \hat{k}) = 0$

$$\Rightarrow 10(10\lambda - 3) - 7(-7\lambda + 3) + 1(\lambda - 4) = 0$$

$$\Rightarrow 150\lambda = 75 \Rightarrow \lambda = \frac{1}{2}$$

$$|\vec{PA}| = \sqrt{(10\lambda - 3)^2 + (-7\lambda + 3)^2 + (\lambda - 4)^2}$$

$$= \sqrt{0 + \left(\frac{1}{2}\right)^2 + \left(\frac{7}{2}\right)^2} = \sqrt{\frac{50}{4}}$$

Which lies in  $(3, 4)$

#1611348

Topic: Plane

If a plane passes through intersection of planes  $2x - y - 4 = 0$  and  $y + 2z - 4 = 0$  and also passes through the point  $(1, 1, 0)$ . Then the equation of plane is

- ☒ A  $x - y - z = 0$
- ☐ B  $2x - z = 0$
- ☐ C  $x + 2z - 1 = 0$
- ☐ D  $x - z - 1 = 0$

**Solution**

$$P_1 + \lambda P_2 = 0$$

$$(2x - y - 4) + \lambda(y + 2z - 4) = 0$$

it passes through  $(1, 1, 0)$

$$\Rightarrow 1 + \lambda = 0 \Rightarrow \lambda = -1$$

equation of plane is  $x - y - z = 0$

#1611349

Topic: Roots and Coefficients

If  $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) = 2$ , then find the sum of roots of equation

- ☐ A 12
- ☐ B 8
- ☐ C 4
- ☒ D 16

**Solution**

$$|\sqrt{x} - 2| + x - 4\sqrt{x} = 2$$

$$\Rightarrow |\sqrt{x} - 2| + (\sqrt{x})^2 - 4\sqrt{x} + 4 = 6$$

$$\Rightarrow (\sqrt{x} - 2)^2 + |\sqrt{x} - 2| = 6$$

$$\text{Let } |\sqrt{x} - 2| = t$$

$$t^2 + t - 6 = 0$$

$$\Rightarrow (t + 3)(t - 2) = 0$$

$$\Rightarrow t = 2 \text{ or } t = -3$$

but  $t = -3$  not possible

$$|\sqrt{x} - 2| = 2$$

$$\Rightarrow \sqrt{x} - 2 = \pm 2$$

$$\Rightarrow \sqrt{x} = 0 \text{ or } \sqrt{x} = 4$$

$$x = 0 \text{ or } x = 16$$

#1611350

Topic: Tangent

$4x^2 + y^2 = 8$ , tangent at  $(1, 2)$  and another tangent at  $(a, b)$  are perpendicular, then find the value of  $a^2$ .

- A**  $\frac{2}{17}$
- B**  $\frac{1}{17}$
- C**  $\frac{8}{17}$
- D**  $\frac{4}{17}$

**Solution**

$$\frac{x'^2}{(\sqrt{2})^2} + \frac{y'^2}{(2\sqrt{2})^2} = 1$$

Let  $(a, b)$  is  $(\sqrt{2} \cos \theta, 2\sqrt{2} \sin \theta)$

tangent at  $(1, 2)$  is  $4x + 2y = 8 \Rightarrow 2x + y = 4 \Rightarrow$  slope is  $-2 = m_1$

tangent at  $(\sqrt{2} \cos \theta, 2\sqrt{2} \sin \theta)$  is  $4\sqrt{2} \cos \theta x + 2\sqrt{2} \sin \theta y = 8 \Rightarrow$  slope is  $-2 \cot \theta = m_2$

$$\text{Now } m_1 m_2 = -1 \Rightarrow 4 \cot \theta = -1 \Rightarrow \cos \theta = \frac{1}{\sqrt{17}} \text{ or } \frac{-1}{\sqrt{17}}$$

$$\Rightarrow a = \sqrt{\frac{2}{17}} \text{ or } -\sqrt{\frac{2}{17}} \Rightarrow a^2 = \frac{2}{17}$$

#1611352

Topic: Applications of Vector Product

Find the magnitude of projection of vector  $2\hat{i} + 3\hat{j} + \hat{k}$ , on a vector which is perpendicular to the plane containing vectors  $\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i} + 2\hat{j} + 3\hat{k}$ .

- A**  $\frac{\sqrt{3}}{\sqrt{2}}$
- B**  $\frac{\sqrt{2}}{\sqrt{3}}$
- C**  $\frac{4}{\sqrt{3}}$
- D**  $\frac{2\sqrt{2}}{\sqrt{3}}$

**Solution**

Normal vector to the plane to plane containing  $\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i} + 2\hat{j} + 3\hat{k}$  is

$$\vec{n} = (\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{n} = \hat{i} - 2\hat{j} + \hat{k}$$

projection of  $(2\hat{i} + 3\hat{j} + \hat{k})$  on  $\vec{n}$

$$= \left| \frac{(2\hat{i} + 3\hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{1+4+1}} \right|$$

$$= \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$