#1611041

Topic: Integration by Substitution

If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = x \cdot f(x) \cdot (1+x^6)^{1/3} + C$, then $f(x)$ is equal to?

A (boxed) $\frac{-1}{2x^3}$

B $\frac{-1}{2x^2}$

C $\frac{-1}{6x^2}$

D $\frac{1}{6x^2}$

Solution

$$\int \frac{dx}{x^3(1+x^6)^{2/3}} \Rightarrow \int \frac{dx}{x^7\left(1+\frac{1}{x^6}\right)^{2/3}} \Rightarrow \left(1+\frac{1}{x^6}\right)^{1/3} = t$$

$$-\frac{6}{x^7}dx = dt$$

$$\Rightarrow -\frac{1}{6}\int \frac{dt}{t^{2/3}}$$

$$-\frac{1}{6}\left(\frac{t^{1/3}}{\frac{1}{3}}\right) = -\frac{1}{2}\left[\left(1+\frac{1}{x^6}\right)^{1/3}\right] + C = -\frac{1}{2}\frac{(1+x^6)^{1/3}}{x^2} + C = x\, f(x)(1+x^6)^{1/3}$$

$$f(x) = -\frac{1}{2x^3}.$$

#1611042

Topic: Differentiation of Implicit Functions

If $f(1) = 1$, $f'(1) = 3$, then the value of derivative of $f(f(fx))) + (f(x))^2$ at $x = 1$ is?

A 9

B (boxed) 33

C 12

D 20

Hint Solution

$$y = f(f(f(x)) + (f(x))^2$$

$$\frac{dy}{dx} = f'(f(f(x)) \cdot f'(f(x)) \cdot f'(x) + 2f(x)f'(x)$$

Put $x = 1 \Rightarrow f'(f(f(1))) \cdot f'(f(1)) \cdot f'(1) + 2f(1) \cdot f'(1) = 27 + 6 = 33.$

#1611044

Topic: Basic Relation Between Sides and Angles

If sides of triangle are in A.P. and the largest angle is double smallest angle then find ratio of sides.

A $3:5:6$

B $4:5:6$

C $2:3:5$

D $3:4:5$

Solution
Hint

Let a, b, c are the sides in increasing order $2b = a + c$

Let angles are

$A = \theta, B = \pi - 3\theta$ and $C = 2\theta$

Now $2\sin B = \sin A + \sin C$

$2\sin 3\theta = \sin\theta + \sin 2\theta$

$2(3 - 4\sin^2\theta) = 1 + 2\cos\theta$

$6 - 8(1 - \cos^2\theta) = 1 + 2\cos\theta$

$8\cos^2\theta - 2\cos\theta - 3 = 0$

$(2\cos\theta + 1)(4\cos\theta - 3) = 0$

$\cos\theta = \frac{3}{4}, \cos\theta = -\frac{1}{2}$ (rejected)

the ratio of sides a : b : c

$\sin A : \sin B : \sin C$

$\sin\theta : \sin 3\theta : \sin 2\theta$

$1 : 3 - 4\sin^2\theta : 2\cos\theta$

$1 : \frac{5}{4} : \frac{6}{4} = 4 : 5 : 6.$

#1611094

Topic: Heights and Distances

Height of two towers are 20m and 80. Join foot of the one tower to the top of other and vice versa. Find the height of intersection point from the horizontal plane.

A 15

B 14

C 16

D 12

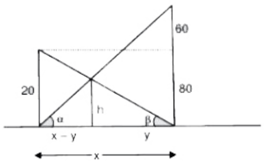
Solution Hint

Height of two towers are 20m c 80m....

$\frac{h}{y} = \tan\beta \Rightarrow \frac{h}{y} = \frac{20}{x}, \frac{h}{x-y} = \frac{80}{x} \Rightarrow \frac{hx}{20} = y, \frac{hx}{80} = x - y$

$\frac{hx}{20} + \frac{hx}{80} = x \Rightarrow \frac{h}{20} + \frac{h}{80} = x \Rightarrow \frac{h}{20} + \frac{h}{80} = 1$

$5h = 80 \ \Rightarrow h = 16\cdot$



#1611104

Topic: L'Hospital's Rule

$f^{'}(3) + f^{'}(2) = 0$. Find the $\lim_{x \to 0} \left(\frac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)}\right)^{\frac{1}{x}}$.

A e^x

B e^2

C 1

D $e^{1/2}$

Solution Hint

$\lim_{x \to 0} \left(\frac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)}\right)^{\frac{1}{x}} \Rightarrow e^{\lim_{x \to 0} \left(\frac{1 + f(3+x) - f(3) - 1 - f(2-x) + f(2)}{x(1 + f(2-x) - f(2))}\right)}$

$e^{\lim_{x \to 0} \frac{f(3+x) - f(2-x) - (f(3) - f(2))}{x(1 + f(2-x) - f(2))}} \Rightarrow e^{\lim_{x \to 0} \frac{f^{'}(3+x) + f^{'}(2-x)}{f}}$

$e^{f^{'}(3) + f(2)} = e^0 = 1\cdot$

#1611105

Topic: ArithmeticoGgeometric Progression

$\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to?

A $2 - \frac{11}{2^{19}}$

B $1 - \frac{11}{2^{20}}$

C $2+\frac{11}{2^{19}}$

D $1+\frac{11}{2^{20}}$

Hint
Solution

$$S=\sum_{k=1}^{20}\frac{k}{2^k}$$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^2}+....+\frac{20}{2^{21}}$$

$$\frac{S}{2}=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^{20}}-\frac{20}{2^{21}}=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{\left(\frac{1}{2}\right)}=\frac{20}{2^{21}}$$

$$S=2\left(1-\frac{1}{2^{20}}\right)-\frac{20}{2^{20}}=2-\frac{22}{2^{20}}=2-\frac{11}{2^{19}}.$$

#1611106

Topic: Nature of Roots

If $(1+m^2)x^2-2(1+3m)x+(1+8m)=0$. Then numbers of value(s) of m for which this equation has no solution.

A 3

[B] ∞

C 2

D 1

Hint
Solution

$D=4(1+3m)^2-(1+m^2)(1+8m)$

$=4(1+9m^2+6m-(1+8m+m^2+8m^3))$

$=4(8m^2-2m-8m^3)$

$=-8(4m^3-4m^2+m)$

$=-8m(4m^2-4m+1)$

$=-8m(2m-1)^2<0$

Hence infinitely many values.

#1611107

Topic: Basics of Straight Lines

If points (h, k) (1, 2) and $(-3,4)$ lie on line L_1 and points (h, k) and (4, 3) lie on L_2. If L_2 is perpendicular to L_1, then value of $\frac{h}{k}$ is?

A $-\frac{1}{7}$

B $\frac{1}{3}$

[C] 3

D 7

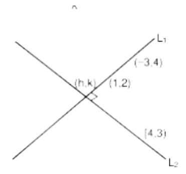
Solution

$L_1 : (y - 2) = -\frac{1}{2}(x - 1) = x + 2y - 5 = 0$

$L_2 : (y - 3) = 2(x - 4) = 2x - y - 5 = 0$

put h, k in both lines

$(h, k) = (3, 1) \Rightarrow \frac{h}{k} = 3.$



#1611357

Topic: Chords of Ellipse

One of the focus of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\left(0, 5\sqrt{3}\right)$ and difference in lengths of major and minor axis is 10 units. Then length of latus rectum is?

A 3

B 5

C 10

D 15

Solution

$be = 5\sqrt{3} \Rightarrow b^2e^2 = 75$

$b^2 - a^2 = 75$

$(b - a)(b + a) = 75$

$b + a = 15$

$b = 10, a = 5$

$LR = 2 \times \frac{a^2}{b} = \frac{2 \times 25}{10} = 5.$

#1611361

Topic: Tangent

The tangent of parabola $y^2 = 4x$ at the point where it cut the circle $x^2 + y^2 = 5$. Which of the following point satisfies the equation of tangent.

A (7, 5)

B (3, 4)

C (2, 2)

D (− 3, − 4)

Solution

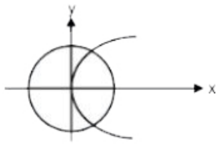
$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5$ and $x = 1$

required point in quadrant first is $(1, 2)$

required equation is $x - y + 1 = 0$ and now check option.



#1611365

Topic: Concurrent and Family of Lines

If points $P(3, 2, 1)$, $R(4, y, z)$, $4Q(2, -1, 3)$ lie on the same line, then distance of point R from origin is?

A $\sqrt{21}$

B $2\sqrt{20}$

C $\sqrt{42}$

D $\sqrt{31}$

Solution

$PQ: \dfrac{x-3}{1} = \dfrac{y-2}{3} = \dfrac{z-1}{-2}$

R and PQ

$\dfrac{4-3}{1} = \dfrac{y-2}{3} = \dfrac{z-1}{-2}$

$R(4, 5, -1)$

$\therefore OR = \sqrt{16+25+1} = \sqrt{42}.$

#1611388

Topic: Euler Form of Complex Number

The value of $(1 + iz + z^5 + z^8)^9$ when $z = \dfrac{\sqrt{3}+i}{2}$ is?

A 0

B −1

C $-i$

D $(-1-i)^9$

Solution

$$\left(1 + e^{i\frac{\pi}{2}} e^{i\frac{\pi}{6}} + e^{i\frac{5\pi}{6}} + e^{i\frac{4\pi}{6}}\right)^9$$

$$\left(1 + e^{i\frac{2\pi}{3}} + e^{i\frac{5\pi}{6}} + e^{i\frac{4\pi}{3}}\right)^9$$

$$\left(1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i - \frac{\sqrt{3}}{2} + \frac{i}{2} - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^9$$

$$\left(\frac{-\sqrt{3}+i}{2}\right)^9 = \left(e^{i\frac{5\pi}{6}}\right)^9 = e^{i\frac{15\pi}{2}} = -i.$$

#1611391

Topic: Properties of Determinants

If $A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$ and $|A| \in [2, 16]$. 2, b, c are in A.P. the range of c is?

A $[2, 4]$

B $[2 + 2^{1/3}, 4]$

C $[3, 2 + 2^{1/3}]$

D $[4, 6]$

Solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & (b-2) & (c-2) \\ 4 & (b^2-4) & (c^2-4) \end{vmatrix} = \begin{vmatrix} (b-2) & (c-2) \\ (b^2-4) & (c^2-4) \end{vmatrix}$$

$$= (b-2)(c-2)\begin{vmatrix} 1 & 1 \\ (b+2) & (c+2) \end{vmatrix}$$

$|A| = (b-2)(c-2)(c-b)$

2, b, c are in A.P.

$2, 2+d, 2+2d$

$|A| = d(2d)d = 2d^3 \in [2, 16] \Rightarrow d^3 \in [1, 8] \Rightarrow d \in [1, 2] \Rightarrow 2d \in [2, 4]$

$2 + 2d \in [4, 6]$.

#1611393

Topic: Permutations

The number of 4 digit numbers that can be formed using digits $0, 1, 2, 3, 4, 5$(repetition allowed) which are greater than 4321 is?

A 306

B 310

C 288

D 280

Solution

Total ways $= 4 + 18 + 72 + 216 = 94 + 216 = 310.$

#1611394

Topic: Probability Distribution

A coin is rolled n times. If the probability of getting head at least once is greater than 90% then the minimum value of n is?

A 4

B 3

C 5

D 6

Solution

$1 - \frac{1}{2^n} > \frac{9}{10} \Rightarrow \frac{1}{10} > \frac{1}{2^n} \Rightarrow 2^n > 10$

$\therefore$ minimum value of n is 4.

#1611399

Topic: Area of Bounded Regions

Let $f(x, y) = \{(x, y) : y^2 \le 4x, 0 \le x \le \lambda\}$ and $s(\lambda)$ is area such that $\frac{S(\lambda)}{S(4)} = \frac{2}{5}$. Find the value of λ.

A $4\left(\frac{4}{25}\right)^{1/3}$

B $4\left(\frac{2}{25}\right)^{1/3}$

C $2\left(\frac{4}{25}\right)^{1/3}$

D $2\left(\frac{2}{25}\right)^{1/3}$

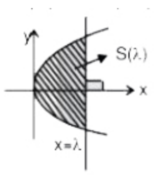
Solution

$y^2 = 4x$

$S(\lambda) = 2\int_0^\lambda 2\sqrt{x}dx = \left.\frac{4x^{3/2}}{3/2}\right]_0^\lambda = \frac{8}{3}\lambda^{3/2}$

$\frac{S(\lambda)}{S(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$

$\lambda = 4\left(\frac{2}{5}\right)^{2/3} = 4\left(\frac{4}{25}\right)^{1/3}$.



#1611400

Topic: Variance and Standard Deviation

The marks of a student in 6 tests are 41, 45, 54, 57, 43 and x. If the mean marks of these tests is 48, then standard deviation of these tests is?

A $\frac{10}{\sqrt{3}}$

B $\frac{10}{\sqrt{2}}$

C $\frac{10}{3}$

D $\frac{20}{3}$

Solution

$\frac{41 + 45 + 54 + 57 + 43 + x}{6} = 48 \Rightarrow x = 48$

$\sigma^2 + 48^2 = \frac{1}{6}(41^2 + 45^2 + 54^2 + 57^2 + 43^2 + 48^2)$

$\sigma^2 = \frac{14024}{6} - 2304 = \frac{7012}{3} - 2013 = \frac{7012 - 6912}{3} = \frac{100}{3}$

$\sigma = \frac{10}{\sqrt{3}}$.

#1611423

Topic: Maxima and Minima

The height of the cylinder of maximum volume which can be inscribed in a sphere of radius 3cm is?

A $\sqrt{3}$

B $2\sqrt{3}$

C $\frac{2\sqrt{3}}{3}$

D $3\sqrt{2}$

Hint Solution

$h = 2(3\cos\theta)$

$r = 3\sin\theta$

$v = \pi r^2 h$

$= \pi 9\sin^2\theta \cdot 6\cos\theta$

$V = 54\pi\sin 2\theta\cos\theta$

$\frac{dv}{d\theta} = 0$

$\Rightarrow 2\sin\theta\cos^{\theta} - \sin^3\theta = 0$

$\Rightarrow 2s(1 - s^2) - s^3 = 0$

$\Rightarrow 2s - 2s^3 - s^3 = 0$

$\Rightarrow 2s - 3s^3 = 0$

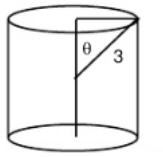
$\Rightarrow s = 0$ or $2 - 3s^2 = 0$

$s = \pm\sqrt{\frac{2}{3}}$

$\therefore \cos\theta = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$

$h = 6\left(\frac{1}{\sqrt{3}}\right)$

$h = 2\sqrt{3}.$



#1611460

Topic: Applications on Geometrical Figures

The area of triangle formed by tangent and normal at point $(\sqrt{3}, 1)$ of the curve $x^2 + y^2 = 4$ and x-axis is?

A $\frac{4}{\sqrt{3}}$

B $\frac{2}{\sqrt{3}}$

C $\frac{8}{\sqrt{3}}$

D $\frac{5}{\sqrt{3}}$

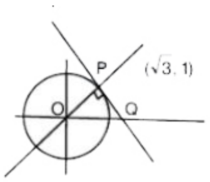
Solution

Slope of OP $= \frac{1}{\sqrt{3}}$, slope of PQ $= -\sqrt{3}$

$y - 1 = -\sqrt{3}(x - \sqrt{3}) = -\sqrt{3}x + 3$

$\Rightarrow \sqrt{3}x + y = 4$ and $Q\left(\frac{4}{\sqrt{3}}, 0\right)$

$\Delta OPQ = \frac{2}{\sqrt{3}}$.



#1611469

Topic: Applications on Geometrical Figures

If the slope of tangent at point (x, y) of curve $y = f(x)$ is given by $\frac{2y}{x^2}$. If this curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$. Then the curve is?

A $x\ln(y) = 2(x - 1)$

B $x^2\ln(y) = 2(x - 1)$

C $x^2\ln(y) = (x - 1)$

D $x\ln(y) = (x - 1)$

Solution

$\frac{dy}{dx} = \frac{2y}{x^2} \Rightarrow \ln y = -\frac{2}{x} + \ln C$

passes through $(1, 1)$

$0 = \frac{-2}{1} + \ln C, \ln C = 2$

$\ln y = -\frac{2}{x} + 2$

$x\ln(y) = 2(x - 1)$.

#1611476

Topic: Truth Tables

Which of the following is not a tautology?

A $p \rightarrow (p \vee q)$

B $(p \wedge q) \rightarrow p$

C $(p \vee q) \rightarrow (p \wedge (\sim q))$

D $(p \vee \sim p)$

Solution

(1) $p \to (p \vee q)$

$\sim p \vee (p \vee q) = t$

(2) $\sim (p \wedge q) \vee P$

$(\sim p \vee \sim q) \vee p = t$

(3) $\sim (p \vee q) \vee (p \wedge \sim q)$

$= (\sim p \wedge \sim q) \vee (p \wedge \sim q)$

$= (\sim p \vee p) \wedge \sim q$

$= t \wedge \sim q = \sim q \neq t$

(4)

Alter:

(a)

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(b)

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(c)

p	q	$p \vee q$	$\sim q$	$p \wedge \sim q$	$p \vee q \to p \wedge (\sim q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	F	F	T
F	F	F	T	F	T

#1611483

Topic: Common Roots

If a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0 = 0$ and $dx^2 + 2ex + f = 0$ have a common root. Then?

A d, e, f are in G.P.

B d, e, f are in A.P.

C $\frac{a}{d}, \frac{b}{e}, \frac{c}{f}$ are in A.P.

D $\frac{a}{d}, \frac{b}{e}, \frac{c}{f}$ are in H.P.

Solution

$b^2 = ac$

roots of $ax^2 + 2bx + c = 0$ are equal i.e., $-\frac{b}{a}$

$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right) + f = 0$

$db^2 - 2bea + fa^2 = 0$

$dc - 2eb + fa = 0$

divide by ac

$\frac{dc}{ac} - \frac{2eb}{b^2} + \frac{fa}{ac} = 0$

$\Rightarrow \frac{d}{a} - \frac{2eb}{b^2} + \frac{fa}{ac} = 0$

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

#1611488

Topic: Discontinuity of a Function

$f(x)$ is defined as $f(x) = \begin{cases} |x| + [x] & -1 < x < 1 \\ x + |x| & 1 \le x < 2 \\ |x| + [x] & 2 \le x < 3 \end{cases}$ then the number of points of discontinuity of $f(x)$ is?

A 2 (marked)

B 3

C 0

D More than 4

Solution

Interval $f(x) = \begin{cases} x \in [-1, 0) & \Rightarrow & (-x-1) \\ x \in [0, 1) & \Rightarrow & x \\ x \in [1, 2) & \Rightarrow & 2x \\ x \in [2, 3) & \Rightarrow & x + 2 \end{cases}$ at $x = 0, 1$

$f(x)$ is discontinuous.

#1611490

Topic: Plane

Equation of plane passing through line of intersection of planes $x + y + z = 1$ and $2x + 3y + z = 5$ and perpendicular to the plane $x - y - z = 0$ is?

A $\vec{r} \cdot (\hat{j} - \hat{k}) + 3 = 0$

B $\vec{r} \cdot (\hat{j} - \hat{k}) - 3 = 0$ (marked)

C $\vec{r} \cdot (\hat{i} + \hat{k}) + 2 = 0$

D $\vec{r} \cdot (\hat{i} + \hat{k}) - 2 = 0$

Solution

Let the plane is $x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + \lambda) = 1 + 5\lambda$

Now $1 + 2\lambda - 1 - 3\lambda - 1 - \lambda = 0$

$\lambda = -\frac{1}{2}$

Equation of Plane $y - z = 3$.