
#1612007

Topic: Special Functions

Let $f(x)$ satisfy the relation $f(x+y) = f(x).\ f(y)$ for all x, y $\in$ N and $f(1) = 2, \sum_{k=1}^{20} f(a+k) = 16(2^{20}-1)$ then value of a is?

A 1

B 2

[✓ C] 3

D 4

Solution

$f(1) = 2\ \&\ f(x+y) = f(x).\ f(y) \forall x, y, \in N.$

Now, $f(n) = f(n-1+1) = f(n-1).\ f(1)$

$= f(n-2).\ f^2(1)$

$= f(n-3).\ f^3(1)$

.

.

.

$= (f(1))^n$

Hence $f(n) = 2^n \forall n \in N$

Given $f(a+1) + f(a+2) + \ldots\ldots + f(a+20)$

$= 2^{a+1} + 2^{a+2} + \ldots + 2^{a+20}$

$= 2^{a+1}(2^{20}-1) \Rightarrow a = 3.$

#1612066

Topic: Properties of Definite Integral

$\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx$ is equal to?

[✓ A] $\frac{\pi}{4} - \frac{1}{4}$

B $\frac{\pi}{4} + \frac{1}{4}$

C $\frac{\pi}{4} + \frac{1}{2}$

D $\frac{\pi}{4} - \frac{1}{2}$

Solution

$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$

$I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} dx$

$I = \int_0^{\pi/4} (1 - \sin x \cos x) dx$

$I = \frac{\pi}{4} + \frac{1}{4}(\cos 2x)_0^{\frac{\pi}{4}}$

$I = \frac{\pi}{4} - \frac{1}{4}.$

#1612071

Topic: Trigonometric Ratios of Any Angle

$\cos^2 10^o + \cos^2 50^o - \cos 10^o \cos 50^o$ is equal to?

A $\frac{3}{2}$

[✓ B] $\frac{3}{4}$

C $\frac{3}{2}(\cos 20^o + 1)$

D $\frac{3}{4}(\cos 20^o + 1)$

Solution

$\cos^2 10^o + \cos^2 50^o - \cos 10^o \cos 50^o$

$= \frac{1}{2}\{1 + \cos 20^o + 1 + \cos 100^o - \cos 60^o - \cos 40^o\}$

$= \frac{1}{2}[\frac{3}{2} + 2\cos 60^o \cos 40^o - \cos 40^o] = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}.$

#1612077

Topic: Combination

A committee of 11 person is to be made from 8 male and 5 female where m is number of ways of selecting at least 6 male and n is the number of ways of selecting at least 3 female, then?

$\boxed{A}$ $m = n = 78$

B $m = n = 68$

C $m + n = 68$

D $m - n = 8$

Solution

Atleast 6 men

M	W
6	5
7	4
8	3

So, $m = {}^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3$

$= 28 + 40 + 10 = 78$

M	W
8	3
7	4
6	5

So $n = {}^5C_3 \times {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot \frac{C}{6} = 10 + 40 + 28 = 78.$

#1612090

Topic: Integration by Substitution

$\int \sec\frac{2}{3}xcose_c\frac{4}{3}xdx$ is equal to?

A $3\tan\frac{1}{3}x + c$

$\boxed{B}$ $-3\cot\frac{1}{3}x + c$

C $-3\tan\frac{1}{3}x + c$

D $\frac{3}{4}\cot-\frac{1}{3}x + c$

Solution

$I = \int(\sec x)^{2/3} \cdot (cosecx)^{4/3}dx$

$= \int \frac{1}{(\sin x)^{4/3} \cdot (\cos x)^{2/3}}dx$

Multiplying numerator and denominator by $cose_c^2x$, we get

$I = \int \frac{cose_c^2x}{(cotx)^{2/3}}dx$

Let $\cot x = t^3$

$\Rightarrow cose_c^2xdx = -3t^2dt$

Hence $I = -3\int \frac{t^2dt}{t^2} = -3t + C = -3(\cot x)^{1/3} + C.$

#1612101

Topic: Normal

If $y = mx + 7\sqrt{3}$ is normal to $\frac{x^2}{18} - \frac{y^2}{24} = 1$ then the value of m can be?

$\boxed{\text{A}}$ $\frac{2}{\sqrt{5}}$

B $\frac{4}{\sqrt{5}}$

C $\frac{1}{\sqrt{5}}$

D $\frac{2}{\sqrt{3}}$

Hint Solution

$$7\sqrt{3} = \frac{42m}{\sqrt{24 - 18m^2}} \Rightarrow \sqrt{3} = \frac{\sqrt{6}m}{\sqrt{4 - 3m^2}} \Rightarrow 4 - 3m^2 = 2m^2$$

$$m = \frac{2}{\sqrt{5}}.$$

#1612114

Topic: Inverse of a Matrix

Let $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

If $A = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ then $A^{-1} = ?$

A $\begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}$

$\boxed{\text{B}}$ $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

C $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$

D $\begin{bmatrix} 1 & 0 \\ -13 & 1 \end{bmatrix}$

Hint Solution

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdot \cdot \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{n(n-1)}{2} = 78 \Rightarrow n = 13$$

$$A = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$$

so $A^{-1} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}.$

#1612139

Topic: Multiplication Theorem

Probability of hitting a target independently of 4 persons are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{8}$. Then the probability that target is hit, is?

A $\frac{1}{192}$

B $\frac{5}{192}$

$\boxed{\text{C}}$ $\frac{25}{32}$

D $\frac{7}{32}$

Solution

$P(H) = 1 - P(\text{Not Hitting})$

$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{7}{8} = \frac{25}{32}.$

#1612142

Topic: Trigonometric Equations

Let $\theta \in [-2\pi, 2\pi]$ and $2\cos^2\theta + 3\sin\theta = 0$ then sum of all solutions is?

[✓ A] 2π

B 3π

C π

D $\frac{\pi}{3}$

Solution

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$

$\Rightarrow \theta = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Hence sum $= 2\pi$.

#1612145

Topic: Variance and Standard Deviation

Standard deviation of four observations $-1, 0, 1$ and k is $\sqrt{5}$ then k will be?

[✓ A] $2\sqrt{6}$

B 1

C 2

D $\sqrt{6}$

Solution

$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$\Rightarrow 5 = \frac{1 + 0 + 1 + k^2}{4} - \left(\frac{-1 + 0 + 1 + k}{4}\right)^2$

$\Rightarrow 5 = \frac{k^2 + 2}{4} = \frac{k^2}{16}$

$\Rightarrow 80 = 4k^2 + 8 - k^2$

$\Rightarrow 72 = 3k^2$

$\Rightarrow k = 2\sqrt{6}.$

#1612149

Topic: Continuity of a Function

Let $f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} & x \neq \frac{\pi}{4} \\ k & x = \frac{\pi}{4} \end{cases}$ Find k for which $f(x)$ is continuous.

A $-\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D 1

Solution

Since $f(x)$ is continuous at $x = \pi/4$

$$\lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \Rightarrow k = \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$$

$$k = \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{cosec^2 x} = \frac{1}{2}.$$

#1612151

Topic: Bernoulli's Differential Equation

Let $y(x)$ satisfying the differential equation $x\frac{dy}{dx} + 2y = x^2$, given $y(1) = 1$ then $y(x) = ?$

A $\frac{x^2}{4} - \frac{3}{4x^2}$

B $\frac{x^3}{4} + \frac{3}{4x^2}$

C $\frac{x^2}{4} + \frac{3}{4x}$

D $\frac{x^2}{4} + \frac{3}{4x^2}$

Solution

$$\frac{xdy}{dx} + 2y = x^2$$

$$\Rightarrow \frac{dy}{dx} + \frac{2y}{x} = x$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = x^2$$

$$y \cdot x^2 = \int x^3 dx$$

$$yx^2 = \frac{x^4}{4} + C$$

$$1 = \frac{1}{4} + C$$

$$\Rightarrow C = \frac{3}{4}$$

$$y = \frac{x^2}{4} + \frac{3}{4x^2}.$$

#1612154

Topic: Continuity and Differentiability

Let $f(x) = 15 - |x - 10|$ and $g(x) = f(f(x))$ then $g(x)$ is non differentiable is?

A $\{5, 10, 15\}$

B $\{5, 10, 15, 20\}$

C $\{10\}$

D $\{5, 15\}$

Solution

$g(x) = f(15 - |x - 10|)$

$= 15 - |15 - |x - 10| - 10| = 15 - |5 - |x - 10||$

$$= \begin{cases} 15 - |x - 5| & x < 10 \\ 15 - |15 - x| & 10 < x \end{cases}$$

$$= \begin{cases} 10 + x & x < 5 \\ 20 - x & 5 < x < 10 \\ x & 10 < x < 15 \\ 30 - x & 15 < x \end{cases}$$

not differentiable at 5, 10, 15.

#1612166

Topic: Area of Bounded Regions

Find area bounded by the curves $x^2 \le y \le x + 2$.

A $\frac{11}{2}$

B $\frac{7}{2}$

$\boxed{\text{C}}$ $\frac{9}{2}$

D $\frac{5}{2}$

Solution

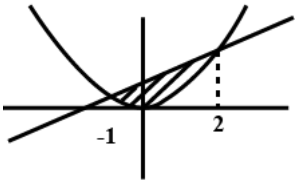
$x^2 \le y \le x + 2$

$x^2 = x + 2$

$\Rightarrow x = -1$ or 2

Hence required area $= \int_{-1}^{2}(x + 2 - x^2)dx$

$$= \frac{x^2}{2} + 2x - \frac{x^3}{3}\Big|_{-1}^{2} = \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}.$$



#1612167

Topic: Multinomial Expansion for Any Real Index

In the expansion of $\left(\frac{2}{x} + x^{log_8 x}\right)^6$ if $T_4 = 20 \times 8^7$ then value of x is?

A $8^{\frac{1}{2}}$

$\boxed{\text{B}}$ 8^2

C 8^3

D 8^4

Solution

$\left(\frac{2}{x} + x^{log_8 x}\right)^6$

$T_4 = {}^6C_3 \cdot \left(\frac{2}{x}\right)^3 \cdot \left(x^{log_8 x}\right)^3 = 20 \times 8^7$

$\Rightarrow \frac{2}{x} \cdot x^{log_8 x} = 2^7 \Rightarrow \frac{x^{log_8 x}}{x} = 2^6 = 8^2$

Taking logarithms on both sides to the base 8, we get

$(log_8 x)^2 = 2 + (log_8 x)$

$\Rightarrow log_8 x = 2$ or -1

$\Rightarrow x = 8^2$ or $\frac{1}{8}$.

#1612169

Topic: Roots and Coefficients

If one root of the quadratic equation $x^2 + px + q = 0$ is $2 - \sqrt{3}$; where p, q $\in$ Q. Then which of the following is true?

A $p^2 - 4q + 12 = 0$

[✓ B] $p^2 - 4q - 12 = 0$

C $q^2 - 4p + 12 = 0$

D $q^2 - 4p - 12 = 0$

Solution

Since p, q $\in$ Q

$\Rightarrow$ other root is $2 + \sqrt{3}$

Hence $p = 4$ and $q = 1$

Hence $p^2 - 4q - 12 = 0$.

#1612172

Topic: Functions

Let the function $f(x)$ defined on $f: R - \{-1, 1\} \to A$ and $f(x) = \frac{x^2}{1 - x^2}$. Find A such that $f(x)$ is surjective.

[✓ A] $R - [-1, 0)$

B $R - [-1, 1)$

C $R - [-1, 2)$

D $R - [0, 1)$

Solution

$f(x) = \frac{x^2}{1 - x^2} = y(2ay)$

$\Rightarrow x^2 y - yx^2 \Rightarrow x^2 = \frac{y}{1 + y} \geq 0$

$\Rightarrow y \in (-\infty, -1) \cup [0, \infty]$

Hence set A should be $R - [-1, 0]$.

#1612176

Topic: Determinants

If α, β are the roots of $x^2 + x + 1 = 0$ then $\begin{vmatrix} y+1 & \beta & \alpha \\ \beta & y+\alpha & 1 \\ \alpha & 1 & y+\beta \end{vmatrix} = ?$

A $y^2 - 1$

B $y(y^2 - 1)$

C $y^2 - y$

$\boxed{\text{D}}$ y^3

Solution

$\Rightarrow \alpha + \beta - 1 \,\&\, \alpha\beta = 1$

Now $R_1 \rightarrow R_1 + R_2 + R_3$ gives

$$\begin{vmatrix} y & y & y \\ \beta & y+\alpha & 1 \\ \alpha & 1 & y+\beta \end{vmatrix}$$

$c_2 \rightarrow c_2 - c_1, c_3 \rightarrow c_3 - c_1$ gives

$$\begin{vmatrix} y & 0 & 0 \\ \beta & y+\alpha+\beta & 1-\beta \\ \alpha & 1-\alpha & y+\beta-\alpha \end{vmatrix} = y\{\{y^2 - (\alpha-\beta)^2\} - (\uparrow - \alpha)(1-\beta)\}.$$

$\Rightarrow y[y^2 - ((\alpha+\beta)^2 - 4\alpha\beta) - 3] \Rightarrow y[y^2 + 3 - 3] = y^3.$

$$x^2+x+1=0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$$

#1612178

Topic: Equation of Line in Parametric Form

If a line is passing through $P(2, 3)$ which intersects the line $x + y = 7$ at a distance of four units from P. Then the slope of line is?

$\boxed{\text{A}}$ $\dfrac{1-\sqrt{7}}{1+\sqrt{7}}$

B $\dfrac{\sqrt{7}-1}{\sqrt{7}+1}$

C $\dfrac{1-\sqrt{5}}{1+\sqrt{5}}$

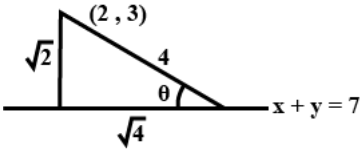
D $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

Solution

$|\tan\theta| = \dfrac{1}{\sqrt{7}} = \left|\dfrac{m-1}{1+m}\right|$

taking + sign $1 + m = m\sqrt{7} - \sqrt{7} \Rightarrow m = \dfrac{\sqrt{7}+1}{\sqrt{7}-1}$

taking − sign $1 + m = \sqrt{7} - \sqrt{7}m \Rightarrow m = \dfrac{\sqrt{7}-1}{\sqrt{7}+1}.$



#1612179

Topic: Tangent and Secant

Find the locus of mid-point of the portion of tangent intercepted between coordinate axes to the circle $x^2 + y^2 = 1$.

$\boxed{\text{A}}$ $x^2 + y^2 - 4x^2y^2 = 0$

B $x^2 + y^2 - 2xy = 0$

C $x^2 + y^2 - 2x^2y^2 = 0$

D $x^2 + y^2 - 16x^2y^2 = 0$

Solution

Let equation of tangent to the given circle be $x\cos\theta + y\sin\theta = 1$

The line meets x-axis at $(\sec\theta, 0)$ & y-axis at $(0, cosec\theta)$. If $P(h, k)$ is the mid-point of this segment.

$\Rightarrow 2h = \sec\theta$ & $2k = cosec\theta$

$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 4$

$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0.$

#1612180
Topic: Arithmetic Progression

Let $a_1, a_2, \ldots\ldots a_{50}$ are non constant terms of an A.P. and sum of n terms is given by $S_n = 50n + (n)(n-7)\frac{A}{2}$, then ordered pair (d, a_{50}) is?(where d is the common difference)

A $(A, 45A)$

[✓ B] $(A, 50 + 46A)$

C $(2A, 46A)$

D $(2A, 50 + 49A)$

Solution

$S_n = 50n + (n)(n-7)\frac{A}{2}$

$a_n = S_n - S_{n-1} = (n-4)A + 50$

$\Rightarrow d = A$

$a_{50} = 46A + 50.$

#1612181
Topic: Chords of Parabola

One end point of a focal chord of a parabola $y^2 = 16x$ is $(1, 4)$. The length of focal chord is?

A 24

[✓ B] 25

C 20

D 22

Solution

Slope $\frac{4-0}{1-4} = \frac{-4}{3} = \tan\alpha$

$L = 4acose_c{}^2\alpha = 16 \times \frac{25}{16} = 25.$

#1612182
Topic: Truth Tables

Find the negation of $pv(\sim p \wedge q)$.

[✓ A] $\sim p \wedge \sim q$

B $\sim p \vee \sim q$

C $p \sim q$

D $p \wedge q$

Solution

$p \vee (\sim p \wedge q) \equiv (p \vee \sim p) \wedge (p \vee q) \equiv p \vee q$

Hence $\sim (p \vee q) \equiv (\sim p \wedge \sim q)$.

#1612183
Topic: Applications on Geometrical Figures

A curve $f(x) = x^3 + ax - b$ pass through $P(1, -5)$ and tangent to $f(x)$ at point P is perpendicular to $x - y + 5 = 0$ then which of the following point will lie on curve?

A $(2, -2)$

B $(2, -1)$

C $(2, -1)$

[✓ D] $(-2, 2)$

Solution

$f(x) = x^3 + ax - b$

It passes through $(1, -5)$ & $f^{'}(1) = -1$

Hence $-5 = 1 + a - b \Rightarrow a - b = -6$

$f^{'}(x) = 3x^2 + a \Rightarrow -1 = 3 + a \Rightarrow a = -4$ Hence $b = 2$

So, $f(x) = x^3 - 4x - 2 \Rightarrow (2, -2)$ lies on it.

#1612184
Topic: Plane

A plane passes through the point $(0, -1, 0)$ and $(0, 0, 1)$ and makes an angle of $\frac{\pi}{4}$ with the plane $y - z = 0$ then the point which satisfies the desired plane is?

A $(\sqrt{2}, -1, 4)$

B $(\sqrt{2}, 1, 2)$

[✓ C] $(\sqrt{2}, 1, 4)$

D $(\sqrt{2}, 2, 4)$

Solution

$ax + by + cz = 1, -b = 1, c = 1$

$ax - y + z = 1$

$$\frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{2}\sqrt{a^2+2}}$$

$\Rightarrow a^2 + 2 = 4 \Rightarrow a = -\sqrt{2}$

$\Rightarrow$ Plane is $-\sqrt{2}x - y + z = 1$

Clearly $(\sqrt{2}, 1, 4)$ satisfy the plane.

#1612185
Topic: Maxima and Minima

Let $f(x)$ be a non-zero polynomial of degree 4. Extreme points of $f(x)$ are $0, -1, 1$. If $f(k) = f(0)$ then?

[✓ A] k has one rational & two irrational roots

B k has four rational roots

C k has four irrational roots

D k has three irrational roots

Solution

Let $f^{'}(x) = \lambda x(x^2 - 1) \Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$

Now $f(0) = f(k) \Rightarrow \frac{k^4}{4} - \frac{k^2}{2} = 0 \Rightarrow k = 0$ or $\pm\sqrt{2}$

Hence (1).

#1612186
Topic: Plane

Consider a plane $x + 2y + 3z = 15$ and a line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ then find the distance of origin from point of intersection of line and plane.

A $\frac{1}{2}$

[✓ B] $\frac{9}{2}$

C $\frac{5}{2}$

D 4

Solution

Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4} = \lambda \Rightarrow x = 2\lambda + 1, y = 3\lambda - 1, z = 4\lambda + 2$

Now substitution in $x + 2y + 3z = 15$

$\Rightarrow (2\lambda + 1) + 2(3\lambda - 1) + 3(4\lambda + 2) = 15$

$\Rightarrow 2\lambda + 1 + 6\lambda - 2 + 12\lambda + 6 = 15 \Rightarrow 20\lambda + 5 = 15 \Rightarrow \lambda = \frac{1}{2}$

Hence point of intersection is $(2, \frac{1}{2}, 4)$

Hence distance from origin is $\sqrt{4 + \frac{1}{4} + 16} = \sqrt{\frac{81}{4}} = \frac{9}{2}$.

#1612187

Topic: Basic Geometry in Argand Plane

If $S = \left\{\frac{\alpha + i}{\alpha - i}; \alpha \in R\right\}$ then the S lies on?

A A circle with radius $= \sqrt{2}$

B A straight line with slope $= -1$

C A straight line with slope $= 1$

[✓ D] A circle with radius $= 1$

Solution

$x + iy = \frac{(\alpha + i)^2}{\alpha^2 + 1}$

$= \frac{\alpha^2 - 1 + 2\alpha i}{\alpha^2 + 1}$

$x = \frac{\alpha^2 - 1}{\alpha^2 + 1}$ & $y = \frac{2\alpha}{\alpha^2 + 1}$

$x^2 + y^2 = \frac{(\alpha^2 - 1)^2 + 4\alpha^2}{(\alpha^2 + 1)^2}$

$x^2 + y^2 = 1$.

#1612188

Topic: Applications of Vector Product

Let $\vec{\alpha} = 3\hat{i} + \hat{j}, \vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{\beta} = \vec{\beta_1} - \vec{\beta_2}$, such that $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$. Find $\vec{\beta_1} \times \vec{\beta_2}$.

A $\frac{1}{2}(3\hat{i} - 9\hat{j} + 8\hat{k})$

B $\frac{1}{2}(\hat{i} - 3\hat{j} + 4\hat{k})$

[✓ C] $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 10\hat{k})$

D $\frac{3}{2}(3\hat{i} + 9\hat{j} + 10\hat{k})$

Hint

Solution

$$\vec{\beta}_1 = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}|^2}\vec{\alpha} = \frac{5}{10}\vec{\alpha} = \frac{\vec{\alpha}}{2} = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$$

$$\vec{\beta}_2 = \vec{\beta}_1 - \vec{\beta} = (-\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k})$$

$$\vec{\beta}_1 \times \vec{\beta}_2 = \frac{1}{2}(-3\hat{i} + 9\hat{j} + 10\hat{k}).$$