#163550

Topic: Standard Simplifications

If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$ then $k = ?$

A $\frac{2}{3}$

B $\frac{4}{3}$

$\boxed{\text{C}}$ $\frac{8}{3}$

D none

Solution

$\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = 4.1^{4-1} = 4$

$\lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2} = \lim_{x \to k} \frac{x^3 - k^3}{x - k} \cdot \lim_{x \to k} \frac{x - k}{x^2 - k^2}$

$= 3k^{3-1} \frac{1}{2k^{2-1}} = \frac{3}{2}k$

$\therefore 4 = \frac{3}{2}k$

$k = \frac{8}{3}$

#1612359

Topic: Permutation Involving Restrictions

Number of 6 digits number divisible by 11 amde by using the digits $0, 1, 2, 5, 7$ and 9 without repetition is equal to

A 55

$\boxed{\text{B}}$ 60

C 62

D 120

Solution

a	b	c	d	e	f

$9 + 7 + 5 + 2 + 1 + 0 = 24$

$|a + c + e - (b + d + f)| = 0$ or a multiple of 11

and $(a + c + e) - (b + d + f)$ can be

$21 - 3 = 18$ (minimum $b + d + f$)

$20 - 4 = 16$

$17 - 5 = 14$

$18 - 6 = 12$

$17 - 7 = 10$

$16 - 8 = 8$

$15 - 9 = 6$

$14 - 10 = 4$

$13 - 11 = 2$

$12 - 12 = 0$

$a + c + e = 12$

and $b + d + f = 12$

so there is only one possible way

$\{a, c, e\}$ and $\{b, d, d\} = \{7, 5, 0\}$

or $\{a, c, e\}$ and $\{b, d, f\} = \{7, 5, 0\}$

$= \{9, 2, 1\}, \{7, 5, 0\}$

number of ways $= 2.3!.3! - 2!3! = 72 - 12 = 60$

#1612364

Topic: Determinants

Let $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & \sin2\theta & \cos2\theta \\ -\sin2\theta & x & 1 \\ \cos2\theta & 1 & x \end{vmatrix}$, then which of following is/are true?

A $\Delta_1 - \Delta_2 = x^3$

[✓ B] $\Delta_1 + \Delta_2 = -2x^3$

C $\Delta_1 + \Delta_2 = -x^3$

D $\Delta_1 - \Delta_2 = 2x^3$

Solution

$\Delta_1 = x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= -x^3$

Similarly $\Delta_2 = -x^3$

$\therefore \Delta_1 + \Delta_2 = -2x^3$

#1612367

Topic: Arithmetic Progression

Let numbers $a_1, a_2, \dots . a_{16}$ are in AP and $A_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$ then $a_1 + a_5 + a_{12} + a_{16}$ is equal to96

A 36

B 96

[✓ C] 76

D 38

Solution

$a_1, a_2, \ldots\ldots a_{16}$ are in A.P.

$a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$a_1 + a_{16} = a_4 + a_{13} = a_7 + a_{16} = a_5 + a_{12}$

$3(a_5 + a_{12}) = 114$

$a_5 + a_{12} = 38$

$(a_1 + a_5 + a_{12} + a_{16}) = 2(a_5 + a_{12}) = 2 \times 38 = 76$

#1612373

Topic: Binomial Coefficients

In the expansion of $(1 + ax + bx^2)(1 - 3x)^{15}$, if coefficient of x^2 is 0 then order pair (a, b) is equal to

A $(28, 325)$

B $(18, 315)$

$\boxed{\text{C}}$ $(28, 315)$

D $(18, 325)$

Solution

Coefficient of x^2 in $(1 + ax + bx^2)(1 - 3x)^{15}$

$= {}^{15}X_2(-3)^2 + a.\ {}^{15}C_1(-3) + b.\ {}^{15}C_0 = 0$

$\frac{15.14}{2}.\,9 - 3.a.15 + b = 0$

$15 \times 63 - 45a + b = 0$ ....(1)

Coefficient of x^3 in $(1 + ax + bx^2)(1 - 3x)^{15}$

${}^{15}C_3(-3)^3 + a.\ {}^{15}C_2(-3)^2 + b.\ {}^{15}C_1(-3) = 0$

$= \frac{15.14.13}{3 \times 2}.\,3^2 - a.3.\ \frac{15.14}{2} + 15.b = 0$

$7.13.3 - 21a + b = 0$ ...(2)

by using (1) - (2)

$672 - 24a = 0 \Rightarrow a = 28$

Hence $b = 315$

#1612378

Topic: Conjugate and its Properties

Let $z = \frac{(1+i)^2}{a-i}, (a > 0)$ and $|z| = \sqrt{\frac{2}{5}}$ then $\bar{z}$ is equal to

$\boxed{\text{A}}$ $-\frac{1}{5} - \frac{3i}{5}$

B $\frac{1}{5} + \frac{3i}{5}$

C $\frac{3}{5} - \frac{1i}{5}$

D $-\frac{3}{5} + \frac{1i}{5}$

Solution

$|z| = \frac{(\sqrt{2})^2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a^2 + 1 = 10 \Rightarrow a = 3.$

Hence, $z = \frac{(1+i)^2}{3-i}$

$\Rightarrow \bar{z} = \frac{(1-i)^2}{3+i} = \frac{(-2i)(3-i)}{10} = \frac{-1-3i}{5}$

#1612385

Topic: Limits of Special Functions

Value of $\lim_{n \to x} \frac{(n+1)^{1/3} + (n+2)^{1/3} + \ldots. + (2n)^{1/3}}{n^{4/3}}$ is equal to

A $\frac{1}{4}(2^{1/4} - 1)$

B $\frac{3}{4}(2^{1/4} - 1)$

$\boxed{\text{C}}$ $\frac{3}{4}(2^{4/3} - 1)$

D $\frac{3}{4}(2^{4/3} + 1)$

Solution

$$\lim_{n \to \infty}\left\{\left(1 + \frac{1}{n}\right)^{1/3} + \left(1 + \frac{2}{n}\right)^{1/3} + ... + \left(1 + \frac{n}{n}\right)^{1/3}\right\}\frac{1}{n} = \lim_{n \to \infty}\sum_{r=1}^{n}\left(1 + \frac{r}{n}\right)^{1/3}\frac{1}{n}$$

$$= \int_0^1(1 + x)^{1/3}dx = \frac{1}{4/3}.(1 + x)^{4/3}|_0^1 = \frac{3}{4}(2^{4/3} - 1)$$

#1612405

Topic: Homogeneous Differential Equation

If $y(x)$ is satisfy the differential equation $\frac{dy}{dx} - (\tan x - y)\sec^2 x$ and $y(0) = 0$. Then $y\left(-\frac{\pi}{4}\right)$ is equal to

$\boxed{\text{A}}$ $(e - 2)$

B $(2e - 1)$

C $(e^2 - 1)$

D $(e + 2)$

Solution

$\frac{dy}{dx} + y(\sec^2 x) = \tan x\sec^2 x$

$I.F. = e^{\int \sec^2 x dx} = e^{fan\, x}$

solution is

$y(e^{\tan\, x}) = \int e^{\tan\, x}\tan\sec^2 x\, dx$

Put $\tan x = t$

$y(e^{\tan x}) = \int t.\, e^t dt = t.\, e^t - e^t + C = e^t(t - 1) + c$

$ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$

$0 = -1 + c \Rightarrow c = 1$

Hence $y\left(-\frac{\pi}{4}\right) = e - 2$

#1612409

Topic: Solving Quadratic Equation

If α and β are roots of the equation $x^2 + \sin\theta.\, 2\sin\theta = 0$ then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$ is equal to

A $\frac{2^{24}}{(8 + \sin\theta)^{12}}$

$\boxed{\text{B}}$ $\frac{2^{12}}{(8 + \sin\theta)^{12}}$

C $\frac{2^{12}}{(8 - \sin\theta)^{12}}$

D $\frac{1}{2^{24}}$

Solution

$x^2 + x\sin\theta - 2\sin\theta = 0$, has roots α and β

$$\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}} = \frac{\alpha^{12}.\beta^{12}}{(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$$

$$= \frac{(-2\sin\theta)^{12}}{(\sqrt{\sin^2\theta + 8\sin\theta})^{24}} = \frac{2^{12}}{(8 + \sin\theta)^{12}}$$

#1612413

Topic: Lines

From point $P(\beta, 0, \beta)$, (where $\beta \neq 0$) A perpendicular is drawn on line $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$. If length of perpendicular is $\sqrt{\frac{3}{2}}$ then value of β is

$\boxed{\text{A}}$ -1

B -2

C 1

D 2

Solution

PM is perpendicular to given line

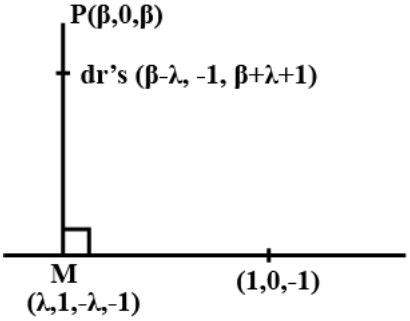
$\Rightarrow \beta - \lambda + 0 - \beta - \lambda - 1 = 0 \Rightarrow \lambda = -\frac{1}{2}$

$M\left(-\frac{1}{2}, 1, -\frac{3}{2}\right)$

$PM = \sqrt{\frac{3}{2}}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 + 1 + \left(\beta + \frac{3}{2}\right)^2 = \frac{3}{2} \Rightarrow 2\beta^2 + \beta + \frac{1}{4} + 1 + \beta^2 + 3\beta + \frac{9}{4} = \frac{3}{2} \Rightarrow 2\beta^2 + 4\beta + 2 = 0$

$(\beta + 1)^2 = 0 \Rightarrow \beta = -1$



#1612414

Topic: Tangent and Secant

A circle is tangent to the line $y = x$ at point $P(1, 1)$ and passes through point $(1, -3)$. Find the radius of the circle.

A $\sqrt{2}$

$\boxed{\text{B}}$ $2\sqrt{2}$

C $\frac{1}{\sqrt{2}}$

D $3\sqrt{2}$

Solution

Family of circle touching a given line at a given point

$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

passes through $(1, -3)$

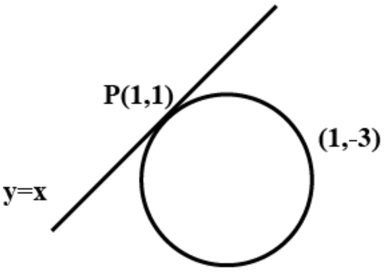
so $0 + 16 + \lambda(1+3) = 0 \Rightarrow \lambda = -4$

so required circle

$x^2 + 1 - 2x + y^2 + 1 - 2y - 4x + 4y = 0$

$x^2 + y^2 - 6x + 2y + 2 = 0$

$r = \sqrt{9 + 1 - 2} = 2\sqrt{2}$



#1612418

Topic: Chords of Circle

If common chord of circle $^2 + y^2 + 5kx + 2y + k = 0$ and $x^2 + y^2 + kx + \frac{y}{2} + \frac{1}{2} = 0$ is $4x - 5y - k = 0$ then number of values of k is

A 0

B 1

[✓ C] 2

D 3

Solution

equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow 4k + \frac{3y}{2} + k - \frac{1}{2} = 0$

Which is identical to

$4x - 5y - k = 0$

Hence $\frac{4k}{4} = \frac{3/2}{-5} = \frac{k - 1/2}{-k}$

$k = -\frac{3}{10}$ and $k62 + \frac{k-1}{2} = 0$

$2k^2 + 2k - 1 = 0$

$k = \frac{+2 \pm \sqrt{4+8}}{4}$

$k = \frac{-1 \pm \sqrt{3}}{2}$

There is no value of k which satisfy simultaneously

#1612421

Topic: Area of Bounded Regions

Region formed by $|x - y| \le 2$ and $|x + y| \le 2$ is

A Rhombus of side is 2

B Square of area is 6

C Rhombus of area is $8\sqrt{2}$

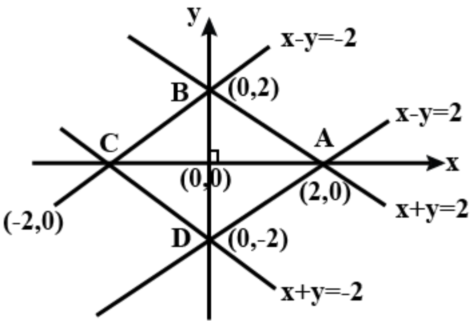
[✓ D] Square of side is $2\sqrt{2}$

Solution

ABCD is a square

$Area = 4 \times \frac{1}{2} \times 2 \times 2 = 8$

side $= 2\sqrt{2}$



#1612425

Topic: Continuity of a Function

If $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \dfrac{\sqrt{x^2 + x} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$ is continuous at $x = 0$ the (p, q) is

A $\left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$

B $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$

C $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$

$\boxed{\text{D}}$ $\left(-\dfrac{3}{2}, \dfrac{1}{2}\right)$

Solution

$f(0^-) = f(0) = f(0^+)$

$$\lim_{h \to 0} \frac{\sin(p+1)(-h) - \sin h}{-h} = q = \lim_{h \to 0} \frac{\sqrt{h^2, + h} - \sqrt{h}}{h\sqrt{h}}$$

$$\lim_{h \to 0} \frac{(p+1)\sin(p+1)h}{(p+1)h} + \frac{\sin h}{h} = p + 1 + 1 = q = \lim_{h \to 0} \frac{\sqrt{h+1} - 1}{h} = \lim_{h \to 0} \frac{h + 1 - 1}{h(\sqrt{h+1} + 1)}$$

$$p + 2 = q = \frac{1}{2} \Rightarrow p = -\frac{3}{2}. q = \frac{1}{2}$$

#1612434

Topic: Probability

There are two family each having two children. If there are at least two girls among the children, find the probability that all children are girls

A $\dfrac{1}{9}$

B $\dfrac{1}{10}$

$\boxed{\text{C}}$ $\dfrac{1}{11}$

D $\dfrac{1}{12}$

Solution

$$\begin{array}{cccccc} & A & & & & B \\ G & G & G & G & \to & 1 \\ G & G & G & B & \to & {}^4C_1 \\ G & G & G & B & \to & {}^4C_2 \end{array}$$

Required probability $= \dfrac{1}{1 + {}^4C_1 + {}^4C_2} = \dfrac{1}{11}$

#1612440

Topic: Heights and Distances

There are three points A, B and C on a horizontal plane; such that $AB = AC = 100m$. A vertical tower is placed on the midpoint of BC; such that angle of elevation of the top of the tower from A is $\cot^{-1}(3\sqrt{2})$. and that from B is $cosec^{-1}(2\sqrt{2})$. then the height of the tower is

A 25

B 10

C $\dfrac{100}{\sqrt{3}}$

$\boxed{\text{D}}$ 20

Solution

$\cot\theta = 3\sqrt{3}$ ...(i)

equation ....(i)

$$\Rightarrow \frac{\sqrt{100^2 - x^2}}{h} = 3\sqrt{2}$$

$$\Rightarrow 100^2 - x^2 = 18h^2 \quad \text{...(ii)}$$

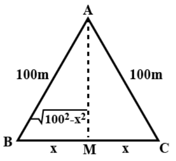
equation (ii)

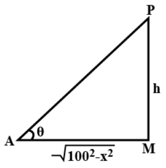
$$\Rightarrow \frac{\sqrt{x^2 + h^2}}{h} = 2\sqrt{2} \Rightarrow x^2 + h = 8h^2 \Rightarrow x^2 = 7h^2 \qquad \text{...(iv)}$$

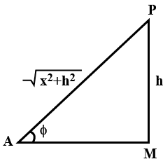
equation (iii) and (iv)

$$100^2 - 7h^2 = 18h^2 \Rightarrow h^2 = \frac{100 \times 100}{25} = 400$$

$h = 20$







#1612445

Topic: Complex Integrations

If $\int \frac{dx}{(x^2 - 2x + 10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2 - 2x + 10}\right)$, then find A and $f(x)$

$\boxed{\text{A}}$ $A = \frac{1}{54}, f(x) = 3(x-1)$

B $A = \frac{1}{54}, f(x) = 9(x-1)^2$

C $A = \frac{1}{27}, f(x) = 9(x-1)^2$

D $A = \frac{1}{81}, f(x) = 3(x-1)$

Solution

$$\int \frac{dy}{(x^2 - 2x + 10)^2} = \int \frac{dx}{))x-1)^2 + 9)^2}$$

Put $x - 1 = 3\tan\theta$

$dx = 3\sec^2\theta d\theta$

$$\Rightarrow \int \frac{3\sec^2\theta d\theta}{(9\sec^2\theta)^2} = \frac{1}{27}\int \frac{d\theta}{sec^2\theta} = \frac{1}{27}\int \cos^2\theta d\theta = \frac{1}{27}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta = \frac{1}{54}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$$

$$= \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2 - 2x + 10}\right] + C$$

#1612448

Topic: Equations of Ellipse

If equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $\left(3, \frac{-9}{2}\right)$ is $x - 2y = 12$. Then length of latus rectum is

$\boxed{\text{A}}$ 9

B $2\sqrt{2}$

C $3\sqrt{2}$

D $2\sqrt{3}$

Solution

$\left(3, -\frac{9}{2}\right)$ lies on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{9}{a^2} + \frac{81}{4b^2} = 1$

Equation of tangent at $\left(3, -\frac{9}{2}\right)$

$\frac{x.3}{a^2} + \frac{y.\left(-\frac{9}{2}\right)}{b^2} = 1$

$\frac{a^2}{3} = 12$ and $\frac{2b^2}{9} = 6$

$a = 6$ and $b = 3\sqrt{3}$

$L.R. = \frac{2b^2}{a} = \frac{2 \times 27}{6} = 9$

#1612452

Topic: Mean

The marks of 20 students in an examination are given in the following table:

Marks	**2**	**3**	**5**	**7**
No. of students	$(x+1)^2$	$2x-5$	x^2-3x	x

Average marks of these student is:

A 2.6

B 2.7

[✓ C] 2.8

D 2.9

#1612462

Topic: Functions

If $f(x) = e^x - x$ and $g(x) = x^2 - x$. Then the interval in which $fog(x)$ is increasing, is

[✓ A] $(0, 1/2) \cup (1, \infty)$

B $(-1/2, 0) \cup (1, \infty)$

C $(-1, \infty)$

D $(-1/2, 0)$

Solution

$h(x) = fog(x) = f(g(x)) = e^{x^2-x} - x^2 + x$

$h'(x) = e(2x-1) - 2x + 1 > 1 > 0 = (2x-1)(e^{x^2-x} - 1) > 0$

Case - I :

$x > \frac{1}{2}$ & $x^2 - x > 0 \Rightarrow x > 1$

Or

Case - II :

$x < \frac{1}{2}$ and $x^2 - x < 0$

$\Rightarrow 0 < x\frac{1}{2}$

So $x \in x \in \left(0, \frac{1}{2}\right) \cup (1, \infty)$

#1612468

Topic: Maths

A hyperbola has center at origin and passing through $(4, -2\sqrt{3})$ and having directrix $5x = 4\sqrt{5}$ then eccentricity of hyperbola (e) satisfy the equation

[✓ A] $4e^4 - 24e^2 + 35 = 0$

B $4e^4 + 24e^2 - 35 = 0$

C $4e^4 - 24e^2 - 35 = 0$

D $4e^4 + 24e^2 + 35 = 0$

#1612482

Topic: Arithmetic Progression

Let $S = 3 + \frac{5(1^3 + 2^3)}{1^2 + 2^3} + \frac{7(1^3 + 2^3 + 3^3}{1^2 + 2^2 + 3^2} + ...$ Then the sum up to 10 terms is

A 220

[✓ B] 660

C 330

D 1320

Solution

General term of given series (T_n)

$$T_n = \frac{(2n + 1)(1^3 + 2^3 + 3^3 ... n)}{(1^2 + 2^2 + ... n)}$$

$$T_n = \frac{(2n + 1)\left(\frac{(n)(n + 1)}{2}\right)^2}{\frac{n(n + 1)(2n + 1)}{6}}$$

$$T_n = \frac{3}{2}n(n + 1) \Rightarrow T_n = \frac{3}{2}[n^2 + n]$$

sum of series $= \sum_{n=1}^{10} T_n = \frac{3}{2}\left[\frac{10(11)(21)}{6} + \frac{10.11}{2}\right]\frac{3}{2}[440] = 660$

#1612501

Topic: Functions

$f(x) = x^2, x \in R$

$S \in [0, 4]$

$g(A) = \{x : x \in R, f(x) \in A\}$ where $A \subset R$

which one is incorrect (where $P \subset Q$ means P is subset of Q)

A $f(g(s)) \neq f(s)$

B $f(g(s)) = f(s)$

[✓ C] $g(f(s)) = g(s)$

D $g(f(s)) \neq s$

Solution

$f(s) = s^2 \qquad 0 \le f(s) \le 16 \qquad ...(i)$

$g(s) = \{x : x \in R, x^2 \in S\}$

$= \{x : x^2 \in [0, 4]\}$

$\Rightarrow \; -2 \le g(s) \le 2 \qquad ...(ii)$

from (i) $0 \le f(s) \le 16$

$g(f(s)) = \{x : f(x) \in f(s)\}$

$= \{x : x^2 \in [0, 16]\}$

$= \{x : -4 \le x \le 4]\} -4 \le g(f(s)) \le 4 \qquad ...(iii)$

from (ii) $-2 \le g(s) \le 2 \Rightarrow 0 \le (g(Ss))^2 \le 4$

$f(g(s)) = g(s)^2$

$0 \le f(g(s)) \le 4 \qquad ...(iv)$

from (iv) and (i) , (1) is true

from (iv) and $S \in [0, 4]$, (2) is true

from (iii) and (ii), (3) is False

from (iii) and $S \in [0, 4]$, (4) is true

so (3) option is correct

#1612505

Topic: Truth Tables

Let (i) $(p \vee q) \vee (p \vee \sim q)$,

(ii) $(p \wedge q) \wedge (p \vee \sim q)$,

(iii) $(p \vee q) \wedge (p \vee \sim q)$,

(iv) $(p \vee q) \vee (p \wedge \sim q)$

which one is tautology

[✓ A] (i)

B (ii)

C (iii)

D (iv)

Solution

(i) $(p \vee q) \vee (p \vee \sin q) = p \vee (q \vee \sin q) = p \vee t = t$

(ii) $(p \wedge q) \wedge (p \vee \sim q)$

(iii) $(p \vee q) \wedge (p \vee \sim q) = p \vee (q \wedge \sim q) - p \vee f = p$

(iv) $(p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim q$	$p \vee q$	$p \wedge \sim q$	$(p \vee q) \vee (p \wedge \sim q)$
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	F	F	F

$(p \wedge q) \wedge (p \vee \sim q)$

p	q	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$(p \wedge q) \wedge (p \vee \sim q)$
T	T	F	T	T	T
T	F	T	F	T	F
F	T	F	F	F	F
F	F	T	F	T	F

#1612508

Topic: Definite Integrals

$\int_0^{2\pi} [\sin 2 \times (1 + \cos 3x)] dx$ (where [·] denotes Greatest Integer Function)

A -2π

B π

C 2π

$\boxed{\text{D}}$ $-\pi$

Solution

$I=\int_0^{2\pi}[\sin2\times(1+\cos3x)]dx$

Apply $a+b-x$

$I=\int_0^{2\pi}[-\sin2\times(1+\cos3x)]dx$

$2I=\int_0^{2\pi}[\sin2\times(1+\cos3x)]+[-\sin2\times(1+\cos3x)]dx$

2I=-2\pi

I=-\pi

#1612510

Topic: Solution of Pair of Equations

Let the equation x + y +z = 5, x + 2y + 2z = 6, x + 3y + \lambda z = \mu have infinite solution then the value of \lambda \mu is10

A 7

$\boxed{\text{B}}$ 10

C 11

D \dfrac{1}{2}

Solution

D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 1 & 3 & \lambda \end{vmatrix}=1.(\lambda-3)

D_1=\begin{vmatrix} 5 & 1 & 1 \\ 6 & 2 & 2 \\ \mu & 3 & \lambda \end{vmatrix}=\begin{vmatrix} 5 & 1 & 1 \\ -4 & 0 & 0 \\ \mu & 3 & \lambda \end{vmatrix}=4.(\lambda-3)

D_2=\begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & \lambda \end{vmatrix}=\begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu -6 & \lambda -2 \end{vmatrix}=\lambda-2-\mu+6=\lambda-\mu+4

D_3=\begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix}=\begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 1 \\ 0 & 1 & \mu -6 \end{vmatrix}=\mu-6-1=\mu-7

For infinitely many solution

D=0,D_1=0,D_2=0,D_3=0

\lambda=3,\lambda=3,\lambda-\mu=-4,\mu=7

\lambda=3 and \mu=7

x+y+z=5.....(i)

x+2y+2z=6...(ii)

x+3y+3z=7....(iii)

from (i) and (ii) y+z=1 \Rightarrow x=4 which stisfy (iii) equation hence there are infinite number of solution \lambda +\mu=10

#1612511

Topic: Properties of Triangles

A (3, 0, -1), B(2, 10, 6) and C(1, 2, 1) are the vertices of a triangle. M is the mid point of the line segment joining AC and G is a point on line segment BM: dividing it in 2:1 ratio internally. Find \cos (\angle GOA).

A \dfrac{2}{\sqrt{2}}

$\boxed{\text{B}}$ \dfrac{1}{\sqrt{15}}

C \dfrac{1}{\sqrt{10}}

D \dfrac{1}{\sqrt{3}}

Solution

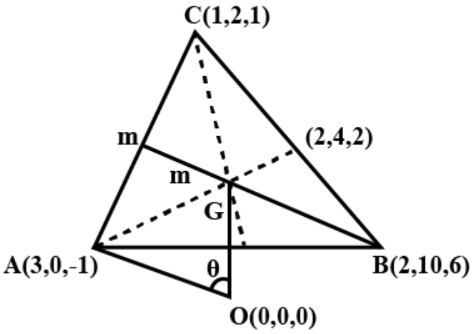
A(3,0-1,),B(2,10,6) and C(1,2,1)

G is centroid of \Delta from given information

\overset{\rightarrow}{OA},\overset{\rightarrow}{OG}=|\overset{\rightarrow}{OA}||\overset{\rightarrow}{OG}|\cos \theta \Rightarrow (2\hat{i}+4\hat{j}+2\hat{k}).(3\hat{i}-\hat{k})=2\sqrt{6}.\sqrt{10}\cos\theta

\Rightarrow 6-2=2\sqrt{3}.\sqrt{5}.2 \cos \theta

\cos \theta =\dfrac{1}{\sqrt{15}}



#1612512
Topic: Triangles

Given a pint P(0, -1, -3) and the image of P in the plane 3x - y 4z - 2 = 0 is Q. Point R is (3, -1, -2). Find the area of \Delta PQR

A \dfrac{\sqrt{91}}{13}

[✓ B] \dfrac{\sqrt{91}}{2}

C \sqrt{\dfrac{91}{2}}

D \sqrt{91}

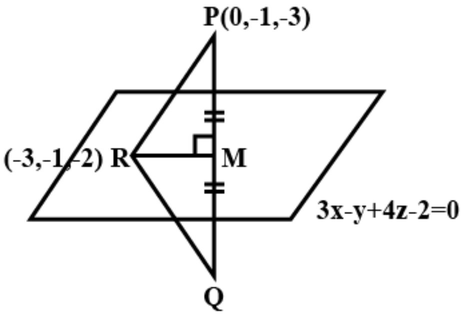
Solution

PM = \left|\dfrac{1-12-2}{\sqrt{9+1+16}}\right| = \sqrt{\dfrac{13}{2}}

PR = \sqrt{9+1} = \sqrt{10}

\therefore RM = \sqrt{10 - \dfrac{13}{2}} = \sqrt{\dfrac{7}{2}}

\therefore \Delta PQR = 2 \times \dfrac{1}{2} \times \sqrt{\dfrac{13}{2}} \times \sqrt{\dfrac{7}{2}} = \dfrac{\sqrt{91}}{2}



#1612513
Topic: Periodicity of Trigonometric Functions

If \dfrac{2^{\sqrt{\sin^2 x-2 \sin x+5}}}{4^{\sin^2 y}} \le 1 then which option is correct

A 2\sin x = \sin y

B |\sin x| = \sin y

[✓ C] \sin x = |\sin y|

D \sin x = 2\sin y

Solution

2^{\sqrt{(\sin x-1)^1 + 4}} \le 4 ^{\sin^2 y}

2\sin^2 y \ge \sqrt{(\sin x- 1)^2 + 4}

\because 2\sin^2y \in [0 , 2]

\sqrt{(2\sin x-1)^2 + 4} \in [2, 2\sqrt{2}]

Hence 2\sin^2y = \sqrt{(\sin x-1)^2 + 4}, for |\sin y | = 1 and \sin x = 1

\Rightarrow |\sin y | = \sin x

#1612520

Topic: Functions

A function f(x) is differnetiable at x = c (c \in R). Let g(x) = |f(x)|, f(c) = 0 then

A g(x) is not differentiable at x =c

[✓ B] for g(x) to be differentiable at c, f'(c) = 0

C for g(x) to be non-differentiable at c, f'(c) = 0

D None of these

Solution

g(x) = |f(x)|

g'(c^+) = \underset{x\to c^+}{\lim} \dfrac{|f(x)|+f(c)}{x-c} = \underset{x \to c^+}{\lim}\dfrac{\pm f(x)}{x-c} = \pm f'(c)

g'(c^-) = \underset{x\to c^-}{\lim}\dfrac{|f(x)|-f(c)}{x-c}

= \underset{x \to c^-}{\lim} \dfrac{\pm f(x)}{x-c}

= \pm f'(c)

for g(x) to be differentiable at x = c.

f'(c) must be 0. Else it is non- differentiable.