#1612446

Topic: Equation of Circles in Different Forms

The locus of centre of circle which touches the circle $x^2 + y^2 = 1$ and y-axis in 1st quadrant is?

A $y = \sqrt{2x - 1}, x > 0$

B $x = \sqrt{2y - 1}, y > 0$

[✓ C] $y = \sqrt{2x + 1}, x > 0$

D $x = \sqrt{2y + 1}, y > 0$

Solution

Let centre is (h, k) & radius is h $(h, k > 0)$

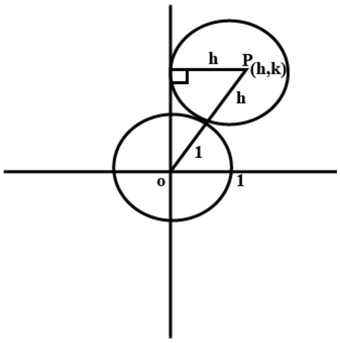
$OP = h + 1$

$\sqrt{h^2 + k^2} = h + 1$

$\Rightarrow h^2 + k^2 = h^2 + 2h + 1$

$\Rightarrow k^2 = 2h + 1$

Locus is $y^2 = 2x + 1$.



#1612455

Topic: Truth Tables

Negation of statement $\sim s \vee (\sim r \wedge s)$ is?

[✓ A] $s \wedge r$

B $s \vee r$

C $\sim s \rightarrow r$

D $\sim s \wedge r$

Solution

$\sim s \vee (\sim r \wedge s)$

$\equiv (\sim s \vee \ \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \ \sim r) \wedge t$

$\equiv \sim s \vee \ \sim r \equiv \sim (s \wedge r)$

Negation of $\sim s \vee (\sim r \wedge s)$ is $s \wedge r$.

#1612458

Topic: Combination

There are 20 pillars of equal height on a circular ground. All pair of non-adjacent pillars are joined by a beam. Then the number of such beams are?

A 180

B 210

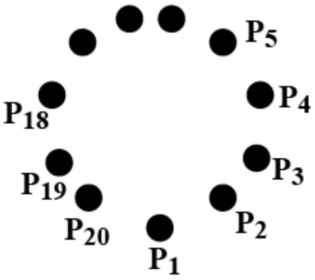
[✓ C] ${}^{20}C_2 - 20$

D ${}^{20}C_2$

Solution

Any two non-adjacent pillars are joined by beams

$\therefore$ number of beams $=$ number of diagonals $= {}^{20}C_2 - 20$.



#1612461

Topic: Maths

If one of the directrix of hyperbola $\frac{x^2}{9} - \frac{y^2}{b} = 1$ is $x = -\frac{9}{5}$. Then the corresponding focus of hyperbola is?

A $(5, 0)$

[✓ B] $(-5, 0)$

C $(0, 4)$

D $(0, -4)$

#1612474

Topic: Conjugate and its Properties

Let z and w be two complex number such that $|zw| = 1$ and $arg(z) - arg(w) = \pi/2$, then?

A $z\bar{w} = -i$

[✓ B] $z\bar{w} = i$

C $z\bar{w} = \frac{1-i}{\sqrt{2}}$

D $z\bar{w} = \frac{1+i}{\sqrt{2}}$

Solution

Let $|z| = r \therefore z = re^{i\theta}$

$|w| = \frac{1}{r} \therefore w = \frac{1}{r}e^{i\phi}$

arg $z - arg\ w = \frac{\pi}{2}$

$\theta - \phi = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} + \phi$

$z\bar{w} = re^{i\theta} \cdot \frac{1}{r}e^{-i\phi}$

$= e^{i\left(\frac{\pi}{2} + \phi\right)} \cdot e^{-i\phi} = i.$

#1612477

Topic: Distance of Point from a Line

A straight line parallel to the straight line $4x - 3y + 2 = 0$ is at a distance of $\frac{3}{5}$ units from the origin. Then which of the following points lie on this line?

A $\left(\frac{1}{4}, \frac{2}{3}\right)$

[✓ B] $\left(-\frac{1}{4}, \frac{2}{3}\right)$

C $\left(\frac{1}{4}, \frac{-1}{3}\right)$

D $\left(-\frac{1}{4}, \frac{-2}{3}\right)$

Solution

Straight line parallel to $4x - 3y + 2 = 0$ is $4x - 3y + \lambda = 0$ whose distance from $(0, 0)$ is $\frac{3}{5}$

$\therefore \left|\frac{\lambda}{5}\right| = \frac{3}{5}$

$\therefore \lambda = \pm 3$

$\therefore$ Straight lines are $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$

$\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfies the first equation.

#1612481

Topic: Definite Integrals

Value of $\int_{\pi/6}^{\pi/3} \sec^{2/3}x \cdot cosec^{4/3}xdx$ is?

A $2\frac{7}{6} - 2\frac{5}{6}$

B $2\frac{5}{6} - 2\frac{3}{4}$

C $3\frac{5}{6} - 3\frac{3}{4}$

$\boxed{\text{D}}$ $3\frac{7}{6} - 3\frac{5}{6}$

Solution

$\int_{\pi/6}^{\pi/3} \sec^{2/3}x \cdot cosec^{4/3}xdx$

$\int_{\pi/6}^{\pi/3} \frac{dx}{\frac{\sin^{4/3}x}{\cos^{4/3}x} \cdot \cos^{2/3}x\cos^{4/3}x}$

$\int_{\pi/6}^{\pi/3} \frac{\sec^2 xdx}{\tan^{4/3}x}$

$\tan x = t$

$\Rightarrow \sec^2 xdx = dt$

$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{dt}{t^{4/3}} = -3\left(\frac{1}{t^{1/3}}\right)_{1\sqrt{3}}^{\sqrt{3}} = -3\left(\frac{1}{\left(\sqrt{3}\right)^{1/3}} - \left(\sqrt{3}\right)^{1/3}\right)$

$-3\left(\frac{1 - 3^{1/3}}{\left(\sqrt{3}\right)^{1/3}}\right) = 3^{\frac{4}{3} - \frac{1}{6}} - 3^{1 - \frac{1}{6}} = 3\frac{7}{6} - 3\frac{5}{6}.$

#1612483

Topic: Limits of Special Functions

If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then the value of $a + b$ is?

A 1

B -5

C 4

$\boxed{\text{D}}$ -7

Solution

$\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$

For existence of limit

$1 - a + b = 0$

$a - b = 1 \cdot (1)$

$\lim\limits_{x \to 1} \dfrac{x^2 - ax + a - 1}{x - 1}$ (using (1))

$= \lim\limits_{x \to 1} \dfrac{(x^2 - 1) - a(x - 1)}{x - 1}$

$= \lim\limits_{x \to 1} (x + 1 - a)$

$= 2 - a = 5$ (given)

$\therefore a = -3$

$b = -4$

$a + b = 7.$

#1612484

Topic: Mean

If both the standard deviation and mean of data set $x_1, x_2, x_3, \ldots . x_{50}$ are 16. Then the mean of the data seg $(x_1 - 4)^2, (x_2 - 4)^2, (x_3 - 4)^2, \ldots .. (x_{50} - 4)^2$ is?

A 200

B 100

[✓ C] 400

D 1600

Solution

$\sum \dfrac{x_i}{50} = 16$

Variance $= 256$

Variance remains same for $(x_i - 4)$ data set

$\therefore \sigma^2 = \dfrac{1}{50} \sum (x_i - 4)^2 - (16 - 4)^2 = 256 \Rightarrow \dfrac{1}{50} \sum (x_i - 4)^2 = 400$

$\therefore$ Mean of $(x_1 - 4)^2, (x_2 - 4)^2, \ldots . (x_{50} - 4)^2$ is $\dfrac{\sum (x_i - 4)^2}{50} = 400.$

#1612493

Topic: Probability

A coin is tossed n times. If the probability of getting at least one head is atleast 99%, then the minimum value of n is?

A 6

[✓ B] 7

C 8

D 9

Solution

$P(H) = \dfrac{1}{2}$, probability of getting at least one head $= 1 - P$(No head) $\geq .99.$

$\therefore 1 - \dfrac{1}{2^n} \geq \dfrac{99}{100} \Rightarrow \dfrac{1}{2^n} \leq \dfrac{1}{100} \Rightarrow 2^n \geq 100 \Rightarrow n \geq 7.$

$\therefore$ Minimum value of $n = 7.$

#1612503

Topic: Integration by Parts

If $\int x^5 e^{-x^2} dx = g(x) \cdot e^{-x^2} + C$ then the value of $g(-1)$ is?

A $\dfrac{3}{2}$

B $\dfrac{5}{2}$

C $-\frac{5}{2}$

D $\frac{e}{2}$

Solution

Put $x^2 = t$

$2xdx = dt$

$\int t^2 e^{-t}\frac{dt}{2}$

$= \frac{1}{2}[-t^2 \cdot e^{-t} + 2\int te^{-1}dt] + c$

$= \frac{1}{2}[-t^2 \cdot e^{-t} - 2te^{-t} + \int 2e^{-t}dt] + c$

$= \frac{1}{2}(-t^2e^{-t} - 2(te^{-1} + e^{-t})) + c$

$= \frac{-(x^4 + 2x^2 + 2)e^{-x^2}}{2} + c$

$g(x) = \frac{-(x^4 + 2x^2 + 2)}{2}$

$g(-1) = -\frac{5}{2}.$

#1612506

Topic: Linear Differential Equation

The solution of differential equation $\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$ is?

A $y = x^2 + c\cos x$

B $y = 2x^2 - c\cos x$

C $y + x^2 = c\cos x$

D $y + 2x^2 = c\cos x$

Solution

$\frac{dy}{dx} + \tan x \cdot y = 2x + x^2\tan x$

I.F. $= e^{ln(\sec x)} = \sec x$

$y\sec x = \int(2x + x^2\tan x)\sec x dx + c = I1 + c$

$y\sec x = \int 2x\sec x dx + x^2\sec x\tan x dx$

$y\sec x = \int 2x\sec x dx + \left(x^2\sec x - \int 2x\sec x dx + c\right)$

$y\sec x = x^2\sec x + c$

$y = x^2 + c\cos x.$

#1612507

Topic: Locus and Its Equation

Let common tangent to curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$ is $y = -ax + c$ then $|c|$ is equal?

A $\sqrt{2}$

B 1

C $\frac{1}{\sqrt{2}}$

D 2

Solution

Tangent to $x^2 + y^2 = 1$ is $y = mx \pm \sqrt{1 + m^2}$

tangent to $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$

$\Rightarrow 1 + m^2 = \frac{2}{m^2} \Rightarrow m^4 + m^2 - 2 = 0$

$m = \pm 1$

common tangents are $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$

$\Rightarrow c = \pm\sqrt{2}$

$\Rightarrow |c| = \sqrt{2}.$

#1612509

Topic: Tangent

Area of triangle formed by tangent and normal to ellipse $3x^2 + 5y^2 = 32$ at point $(2, 2)$ and x-axis is?

$\boxed{A}$ $\frac{68}{15}$

B $\frac{36}{15}$

C $\frac{32}{3}$

D $\frac{31}{3}$

Solution

$3x^2 + 5y^2 = 32$

$\Rightarrow 6x + 10y\frac{dy}{dx} = 0$

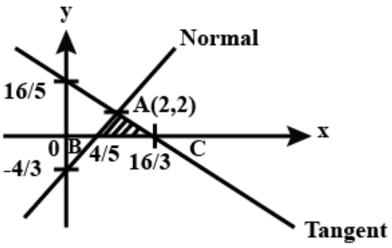
$\Rightarrow \frac{dy}{dx} = -\frac{6x}{10y}$

$\Rightarrow \frac{dy}{dx} = \frac{3x}{5y}$

$m_T = -\frac{3}{5} \Rightarrow$ Equation of tangent is $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow 3x + 5y = 16$

$m_N = 5/3 \Rightarrow$ Equation of normal is $y - 2 = \frac{5}{3}(x - 2) \Rightarrow 5x - 3y = 4$

$A = \frac{1}{2} \times \left(\frac{16}{3} - \frac{4}{5}\right) \times 2 = \frac{68}{15}.$



#1612514

Topic: Quadratic Equations

Area enclosed by curves $y = 2^x$ and $y = |x + 1|$ in the first quadrant is?

A $\frac{1}{2} - \frac{1}{log2}$

B $\frac{3}{2} - \frac{1}{2log2}$

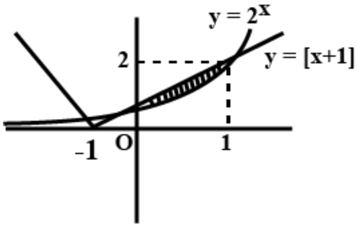
$\boxed{C}$ $\frac{3}{2} - \frac{1}{log2}$

D $\frac{1}{2} + \frac{3}{log2}$

Solution

Area $= \int_0^1(|x+1| - 2^x)dx = \int_0^1(x+1-2^x)dx = \left(\frac{x^2}{2} + x - \frac{2^x}{ln2}\right)_0^t$

$= \left(\frac{1}{2} + 1 - \frac{2}{ln2}\right) - \left(0 + 0 - \frac{1}{ln2}\right) = \frac{3}{2} - \frac{1}{ln2}.$



#1612523

Topic: Solving Quadratic Equation

If the foot of perpendicular drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ on the plane $x + y + z = 3$ also lies on the plane $x - y + z = 3$, then the coordinates of the foot of perpendicular is?

A (− 2, 0, 5)

B (− 1, 0, 4)

C (1, 0, 2)

[✓ D] (2, 0, 1)

Solution

$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1} = \lambda$

$P(2\lambda + 1, \ -\lambda - 1, \lambda)$

foot of perpendicular

$\frac{x-(2\lambda+1)}{1} = \frac{y+(\lambda+1)}{1} = \frac{z-\lambda}{1} = \frac{-(2\lambda+1-\lambda-1+\lambda-3)}{3}$

$\frac{x-(2\lambda+1)}{1} = \frac{y+\lambda+1}{1} = \frac{z-\lambda}{1} = \frac{-(2\lambda-3)}{3}$

$\Rightarrow x = 2\lambda + 1 - \ - \frac{(2\lambda-3)}{3}$

$\Rightarrow y = \ -\lambda - 1 - \ - \frac{(2\lambda-3)}{3} = \frac{-3\lambda-3-2\lambda+3}{3} = -\frac{5\lambda}{3}$

$\Rightarrow z = \lambda - \frac{(2\lambda-3)}{3} = \frac{\lambda+3}{3}$

$\therefore$ point P is $\left(\frac{4\lambda+6}{3}, \frac{-5\lambda}{3}, \frac{\lambda+3}{3}\right)$

It lies on $x - y + z = 3$

$\frac{4\lambda+6}{3} + \frac{5\lambda}{3} + \frac{\lambda+3}{3} = 3 \Rightarrow 10\lambda + 9 = 9 \Rightarrow \lambda = 0$

$\therefore$ point P becomes $(2, 0, 1) \Rightarrow (4)$ option is correct.

#1612530

Topic: Basics of Straight Lines

If three parallel planes are given by

$P_1: 2x - y + 2z = 6$

$P_2: 4x - 2y + 4z = \lambda$

$P_3: 2x - y + 2z = \mu$

If distance between P_1 and P_2 is $\frac{1}{3}$ and between P_1 and P_3 is $\frac{2}{3}$, then the maximum value of $\lambda + \mu$ is?

$\boxed{\text{A}}$ 22

B 20

C 18

D 24

Solution

$P_1: 2x - y + 2z = 6$

$P_2: 4x - 2y + 4z = \lambda$

$P_3: 2x - y + 2z = \mu$

Distance between P_1 and P_2 = $\left|\dfrac{\frac{\lambda}{2} - 6}{3}\right| = \dfrac{1}{3}$

$\therefore \frac{\lambda}{2} - 6 = \pm 1$

$\therefore \lambda = 14, 10$

Distance between P_1 and P_3 = $\left|\dfrac{\mu - 6}{3}\right| = \dfrac{2}{3}$

$\mu - 6 = \pm 2$

$\therefore \mu = 8, 4$

$(\lambda + \mu)_{max} = 22.$

#1612538

Topic: Arithmetic Progression

If terms $a_1, a_2, a_3, \ldots, a_{50}$ are in A.P. and $a_6 = 2$. Then the value of common difference at which maximum value of $a_1 a_4 a_5$ occur is?

A $\frac{3}{5}$

$\boxed{\text{B}}$ $\frac{8}{5}$

C $\frac{2}{5}$

D $\frac{2}{3}$

Solution

$a_1, a_2, a_3, . a_{50}$ in A.P.

$a_6 = 2$

$\therefore a_1 + 5d = 2$

$a_1 a_4 a_5 = a_1(a_1 + 3d)(a_1 + 4d)$

$= a_1(2 - 2d)(2 - d)$

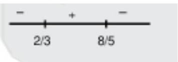
$= -2((5d - 2)(d - 1)(d - 2)$

$= -2(5d^3 - 17d^2 + 16d - 4)$

$\frac{dA}{d(d)} = -2(15d^2 - 34d + 16)$

$= -2(5d - 8)(3d - 2)$

Maximum occurs at $d = \frac{8}{5}$.



#1612539

Topic: Geometric Progression

If a, b, c are in G.P. and $3a, 7b, 15c$ are first 3 terms of A.P. Also the common ratio of G.P. $\in \left(0, \frac{1}{2}\right)$. Then the 4th term of A.P. is?

A $\frac{3}{2}a$

B $\frac{7}{2}a$

C $\frac{5}{2}a$

[✓ D] a

Solution

Let $b = ar, c = ar^2$

Hence $3a + 15ar^2 = 14ar$

$15r^2 - 14r + 3 = 0$

$15r^2 - 9r - 5r + 3 = 0$

$(3r - 1)(5r - 3) = 0$

$r = \frac{1}{3}, \frac{3}{5}$

$\Rightarrow r = \frac{1}{3}$

AP is $3a, \frac{7}{3}a, \frac{5}{3}a, a, \ldots.$

$\Rightarrow 4^{th}$ terms is a.

#1612540

Topic: Properties of Triangles

In a ΔABC, $c = 4$ and angles A, B and C are in A.P. Also ratio $a : b$ is $1 : \sqrt{3}$. Then area of ΔABC is?

A $\sqrt{3}$

[✓ B] $2\sqrt{3}$

C $3\sqrt{3}$

D $4\sqrt{3}$

Solution

$2B = A + C$

$2B = \pi - B$

$3B = \pi$

$B = \frac{\pi}{3}$

$\frac{a}{b} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\sin A}{\sin B} = \frac{1}{\sqrt{3}}$

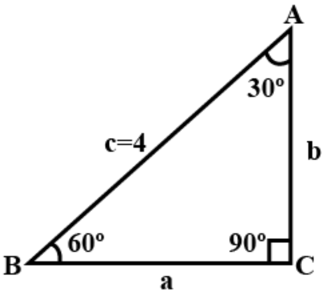
$\Rightarrow \frac{2\sin A}{\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \sin A = \frac{1}{2} \Rightarrow A = 30^o$

$\frac{a}{\sin 30^o} = \frac{b}{\sin 60^o} = \frac{4}{\sin 90^o} = 4$

$a = 4 \times \frac{1}{2} = 2$

$b = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

Area of triangle $= \frac{1}{2}ab = \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}.$



#1612541

Topic: Binomial Expansion for Positive Integral Index

If the coefficient of x in binomial expansion of expression $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$. Then the minimum value of n is?

A 28

B 48

C 58

[✓ D] 38

Solution

$\left(x^2 + \frac{1}{x^3}\right)^n$

$T_{r+1} = {}^nC_r \cdot (x^2)^{n-r}\left(\frac{1}{x^3}\right)^r$

$= {}^nC_r \cdot x^{2n-2r-3r} = {}^nC_r \cdot x^{2n-5r}$

For coefficient of $2n - 5r = 1$

$r = \frac{2n-1}{5}$

Coefficient of x is $= {}^nC_{\frac{2n-1}{5}}$ or ${}^nC_{n-\frac{2n-1}{5}}$ (i.e. ${}^nC_{\frac{3n+1}{5}}$)

$\frac{2n-1}{5} = 23 \Rightarrow 2n = 116 \Rightarrow n = 58$

or $\frac{3n+1}{5} = 23 \Rightarrow 3n + 1 = 115 \Rightarrow n = 38$

Minimum value of n is 38.

#1612542

Topic: Quadratic Equations

Number of real solutions of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is?

A 0

$\boxed{\text{B}}$ 1

C 2

D 3

Solution

$5 + |2^x - 1| = 2^{2x} - 2.2^x$

Case- $1: x \ge 0$

$\Rightarrow 5 + 2^x - 1 = 2^{2x} - 2.2^x$

$\Rightarrow 0 = (2^x - 4)(2^x + 1) \Rightarrow x = 2$

Case-$2: x < 0$

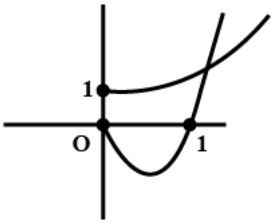
$\Rightarrow 5 + 1 - 2^x = 2^{2x} - 2.2^x$

$\Rightarrow 5 + 1 = 2^{2x} - 2^x$

LHS $= +$ve & RHS $= -$ve

$\therefore \phi$

$\therefore$ Number of solution $= 1$.



#1612543

Topic: Functions

If $f(x) = ln(\sin x)$ and $g(x) = \sin^{-1}(e^{-x})$ for all $x \in (0, \pi)$ and $f(g(\alpha)) = b$ and $(f(g(x))'$ at $x = \alpha$ is a then which is true?

A $a\alpha^2 - b\alpha + 1 = a$

B $a\alpha^2 - b\alpha = -a$

$\boxed{\text{C}}$ $a\alpha^2 - b\alpha + 1 = -a$

D $a\alpha^2 - b\alpha - 1 = -a$

Solution

$f(g(x)) = ln(\sin(\sin^{-1}(e^{-x})))$

$= ln(e^{-x}) = -x$

$(f(g(x))' = -1$

Now $f(g(\alpha) = -\alpha = b$

and $f(g(x))'$ at $x = \alpha$ is $-1 = a$

Now $a\alpha^2 - b\alpha + 1 = -\alpha^2 - (-\alpha)\alpha + 1 = 1 = -a.$

#1612544

Topic: Quadratic Equations

If $\cos^{-1}x - \cos^{-1}(y/2) = \alpha, x \in [-1, 1], y \in [-2, 2]$. Then the value of $4x^2 - 4xy\cos\alpha + y^2$ is?

$\boxed{\text{A}}$ $4\sin^2\alpha$

B $2\sin^2\alpha$

C $4\cos^2\alpha$

D $2\cos^2\alpha$

Solution

$\cos^{-1}\left(\frac{xy}{2}+\sqrt{1-x^2}\cdot\sqrt{1-\frac{y^2}{4}}\right)=\alpha$

$\frac{dy}{2}+\frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2}=\cos\alpha$

$xy+\sqrt{1-x^2}\sqrt{4-y^2}=2\cos\alpha$

$\sqrt{1+x^2}\sqrt{4-y^2}=2\cos\alpha-xy$

$(1-x^2)(4-y^2)=4\cos^2\alpha+x^2y^2-4xy\cos\alpha$

$4-y^2-4x^2+x^2y^2\equiv 4\cos^2\alpha+x^2y^2-4xy\cos\alpha$

$4x^2+y^2-4xy\cos\alpha=4-4\cos^2\alpha=4\sin^2\alpha.$

#1612545

Topic: Sphere

A spherical ball of radius 10cm is enclosed by ice of uniform thickness in spherical shape. If ice melts at the rate of $50cm^3/min$, then the rate of decrease of thickness of ice when thickness of ice is 5cm is?

A $\frac{1}{36\pi}cm/min$

B $\frac{1}{9\pi}cm/min$

$\boxed{\text{C}}$ $\frac{1}{18\pi}cm/min$

D $\frac{2}{9\pi}cm/min$

Solution

$r=10$cm

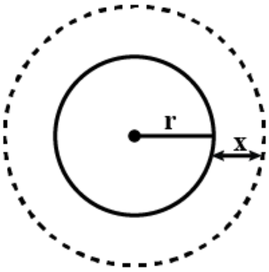
volume of ice $=\frac{4}{3}\pi(r+x)^3-\frac{4}{3}\pi r^3$

$\frac{dv}{dt}=50cm^3/min$

$4\pi(r+x)^2\frac{dx}{dt}=50$

$4\pi(15)^2\frac{dx}{dt}=50$ at $r=10$ and $x=5$

$\frac{dx}{dt}=\frac{50}{4\pi(225)}=\frac{1}{18\pi}cm/min.$



#1612546

Topic: Parallel Lines and Transversal

If a tangent is drawn parallel to the line $6x-18y-11=0$ to the curve $y=\frac{x}{x^2-3}$ which touches the curve at point (α,β), then?

$\boxed{\text{A}}$ $|6\alpha+2\beta|=19$

B $|6\alpha+2\beta|=11$

C $|2\alpha+6\beta|=7$

D $|2\alpha+6\beta|=11$

#1612547

Topic: Special Series

Value of $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\ldots+\frac{1^3+2^3+\ldots\ldots 15^3}{1+2+\ldots\ldots+15}-\frac{1}{2}(1+2+\ldots\ldots+15)$ is?

A 840

B 720

C 680

[✓ D] 620

Solution

$$\sum_{r=1}^{15}\frac{\left(\frac{(n)(n+1)}{2}\right)^2}{\left(\frac{(n)(n+1)}{2}\right)}-\frac{1}{2}\left(\frac{15\times 16}{2}\right)=\sum_{r=1}^{15}\left(\frac{n^2}{2}+\frac{n}{2}\right)-60=620+60-60=620s.$$