#1612948

Topic: Arithmetic Progression

$\left[-\frac{1}{3}\right]-\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots+\left[-\frac{1}{3}+\frac{2}{100}\right]+\ldots.+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is equal to? (where $[.]$ denotes greatest integer function)

A -132

[✓ B] -133

C -134

D -131

Solution

$\left[-\frac{1}{3}\right]+\left[\frac{-1}{3}-\frac{1}{100}\right]+\left[\frac{-1}{3}-\frac{2}{100}\right]+\ldots..+\left[-\frac{1}{3}-\frac{99}{100}\right]$

$=(-1-1-1-\ldots...67$ times $)+(-2-2-2-\ldots.33$ times$)=-133$.

#1612955

Topic: Integration by Parts

$\int\frac{2x^3-1}{x^4+x}dx$ is equal to?

[✓ A] $ln\left|\frac{x^3+1}{x}\right|+c$

B $ln\left|\frac{x^3+1}{x^2}\right|+c$

C $\frac{1}{2}\ln\left|\frac{x^3+1}{x^2}\right|+c$

D $\frac{1}{2}ln\left|\frac{x^3+1}{x}\right|+c$

Solution

$\int\frac{2x^3-1}{x^4+x}dx=\int\frac{2x-x^{-2}}{x^2+x^{-1}}dx=ln(x^2+x^{-1})+c=ln(x^3+1)-lnx+c$

#1612961

Topic: Definite Integrals

If $\int_0^{\pi/2}\frac{\cot x}{\cot x+cosecx}dx=m(\pi-n)$, then the value of mn is?

A $-\frac{1}{2}$

B -1

C $\frac{1}{2}$

[✓ D] 1

Solution

$\int_0^{\pi/2}\frac{\cot x}{\cot x+cosecx}dx=\int_0^{\pi/2}\frac{\cos x}{\cos x+1}dx$

$=\int_0^{\pi/2}\frac{2\cos^2\frac{x}{2}-1}{2\cos^2\frac{x}{2}}dx=\int_0^{\pi/2}\left(1-\frac{1}{2}\sec^2\frac{x}{2}\right)dx$

$=\left(x-\tan\frac{x}{2}\right)_0^{\pi/2}=\frac{\pi}{2}-1=\frac{1}{2}(\pi-2)$

$mn=\frac{1}{2}\times 2=1$

#1612963

Topic: Truth Tables

Let $p\rightarrow(\sim q\vee r)$ is false, then truth values of p, q, r are respectively.

A F, T, T

B T, F, T

[✓ C] T, T, F

D F, F, T

#1612973

Topic: Applications of Dot Product

If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ Then a vector of magnitude 12, which is perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, is?

A $4(2\hat{i} + 2\hat{j} + \hat{k})$

[✓ B] $4(2\hat{i} - 2\hat{j} - \hat{k})$

C $4(\hat{i} - 2\hat{j} - 2\hat{k})$

D $4(2\hat{i} + \hat{j} + 2\hat{k})$

Solution

Required vector is $\vec{r} = \lambda((\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \lambda(16\hat{i} - 16\hat{j} - 8\hat{k})$

$\Rightarrow r = 8\lambda(2\hat{i} - 2\hat{j} - \hat{k})$

$\Rightarrow |\vec{r}| = |8\lambda| \cdot 3$

$\Rightarrow 8\lambda = \pm 4$

$\vec{r} = \pm 4(2\hat{i} - 2\hat{k} - \hat{k})$

#1612983

Topic: Chords of Circle

There are two orthogonal circles with radii 5 and 12 units, then the length of their common chord is?

A $\frac{60}{13}$

[✓ B] $\frac{120}{13}$

C $\frac{30}{13}$

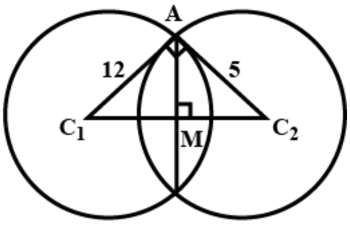
D $\frac{240}{13}$

Solution

$C_1C_2 = \sqrt{12^2 + 5^2} = 13$

Area of $\Delta AC_1C_2 = \frac{1}{2} \cdot 12.5 = \frac{1}{2} \cdot 13\frac{AB}{2}$

$\Rightarrow AB = \frac{120}{13}$ units.



#1612987

Topic: Combination

The coefficient of x^{18} in the expansion of $(1+x)(1-x)^{10}\{(1+x+x^2)^9\}$is?

[✓ A] 84

B 126

C -42

D 42

Solution

Coefficient of x^{18} in $(1+x)(1-x)^{10}(1+x+x^2)^9$

$=$Coefficient of x^{18} in $(1-x)^2\{(1-x)(1+x+x^2)\}^9$

$=$Coefficient of x^{18} in $(1-x^2)(1-x^3)^9$

$= {}^9C_6 = 0 = 84$

#1612990

Topic: Solving Quadratic Equation

Find the number of solutions of the equation $1+\sin^4 x = (\cos 3x)^2$in the interval $\left[\frac{-5\pi}{2}, \frac{5\pi}{2}\right]$.

A 3

B 4

[✓ C] 5

D 6

Solution

$\underbrace{1+\sin^4 x}_{\geq 1} = \underbrace{\cos^2 3x}_{\leq 1}$

Hence for equality to hold $\sin^4 x = 0$& $\cos^2 3x = 1$

$\sin^4 x = 0$

$\Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$

All of which satisfy $\cos^2 3x = 1 \Rightarrow 5$solutions.

#1612993

Topic: Functions

If $f(x) = \tan x, g(x) = \sqrt{x}\, h(x) = \dfrac{1-x^2}{1+x^2}$ and $\phi(x) = (ho(gof))(x)$, then $\phi\left(\dfrac{\pi}{3}\right)$ is equal to?

A $\tan\dfrac{5\pi}{12}$

B $\tan\dfrac{7\pi}{12}$

C $\tan\dfrac{\pi}{12}$

[✓ D] $\tan\dfrac{11\pi}{12}$

Solution

$\phi(x) = (ho(gof)(x) = h(\sqrt{\tan x}) \Rightarrow \phi(x) = \dfrac{1-\tan x}{1+\tan x} = \tan\left(\dfrac{\pi}{4} - x\right)$

$\therefore \phi\left(\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{3}\right) = \tan\left(-\dfrac{\pi}{12}\right) = \tan\dfrac{11\pi}{12}.$

#1612999

Topic: Area of Bounded Regions

Area of the region bounded by $y^2 \leq 4x, x+y \leq 1, x \geq 0, y \geq 0$is $a\sqrt{2}+b$, then value of $a-b$is?

A 4

[✓ B] 6

C 8

D 12

Solution

Let P be the point common to $x+y=1$& $y^2=4x$

So $y^2=4(1-y \Rightarrow y^2+4y-4=0$

$\Rightarrow y=\dfrac{-4\pm\sqrt{16+16}}{2}$

$\Rightarrow=-2+2\sqrt{2}$

Hence $P(3,-2\sqrt{2},-2+2\sqrt{2})$

Hence started area $=$ Area of region (OPN)$+$ Area of (ΔOPQ)

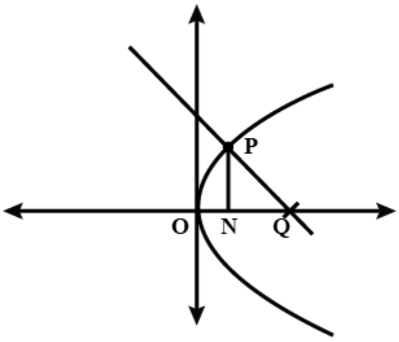
$=\int_0^{3-2\sqrt{2}}2\sqrt{x}dx+\frac{1}{2}[-1-(3-2\sqrt{2})]^2$

$=\frac{2}{3}\cdot 2(\sqrt{2}-1)(3-2\sqrt{2})+\frac{1}{2}[2(\sqrt{2}-1)]^2$

$=\frac{4}{3}\{-7+5\sqrt{2}\}+2(3-2\sqrt{2})=\left(\frac{20}{3}-4\right)\sqrt{2}+6-\frac{28}{3}=\frac{8}{3}\sqrt{2}-\frac{10}{3}$

Hence $a=\frac{8}{3},b=\frac{-10}{3}$

So $a-b=6$



#1613000

Topic: Arithmetic Progression

In an A.P. $S_4=16,S_6=-48$(where S_n denotes the sum of first n term of A.P.), then S_{10} is equal to?

A 320

[✓ B] -320

C 280

D -280

Solution

$S_4=\frac{4}{2}(2a+3d)=16\Rightarrow 2a+3d=8$

$S_6=\frac{6}{2}(2a+5d)=-48\Rightarrow 2a+5d=-16$

$\therefore d=-12$ and $a=22$, Now $S_{10}=\frac{10}{2}(44-108)=-320$Ans.

#1613001

Topic: Equations of Ellipse

For an ellipse $3x^2+4y^2=12$ A normal is drawn to P which is parallel to the line $2x+y=4$ If tangent at P passes through $Q(4,4)$ then length PQ is?

A $5\sqrt{\frac{5}{2}}$

[✓ B] $\frac{5\sqrt{5}}{2}$

C $\frac{5}{2}$

D $5\sqrt{5}$

Solution

Equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$

Normal at $P(2\cos\theta,\sqrt{3}\sin\theta)$is $2x\sin\theta-\sqrt{3}y\cos\theta=\sin\theta\cos\theta$as normal is parallel to $2x+y=4$

$\Rightarrow \frac{2}{\sqrt{3}}\tan\theta=-2$

$\Rightarrow \tan\theta=-\sqrt{3}(1)$

Tangent at $P(2\cos\theta,\sqrt{3}\sin\theta)$is

$\sqrt{3}x\cos\theta+2y\sin\theta=2\sqrt{3}$

Passes through $(4,4)$

$\Rightarrow 4\sqrt{3}\cos\theta+8\sin\theta=2\sqrt{3}(2)$

by (1) & (2)

$\theta=\frac{2\pi}{3}$

$\Rightarrow P\left(-1,\frac{3}{2}\right)$ & $Q(4,4)$

$\Rightarrow PQ=\sqrt{25+\frac{25}{4}}=\frac{5\sqrt{5}}{2}$.

#1613002

Topic: Mean

If $x_1,x_2,......x_{10}$are 10 observations, in which mean of x_1,x_2,x_3,x_4 is 11 while mean of $x_5,x_6,...x_{10}$is 16. Also $x_1^2+x_2^2+....x_{10}^2=2000$then value of standard deviation is?

A 1.5

B 3

C 2.5

[✓ D] 2

Solution

$$\sigma^2=\frac{\sum x_i^2}{10}-\left(\frac{\sum x_i}{10}\right)^2\rightarrow(1)$$

Now $x_1+x_2+x_3+x_4=44$& $x_5+x_6+....+x_{10}=96$

Hence $\sigma^2=\frac{2000}{10}-\left(\frac{140}{10}\right)^2=200-196=4$

Hence $\sigma=2$

#1613004

Topic: Combination

There are 31 objects in a bag in which 10 are identical, then the number of ways of choosing 10 objects from bag is?

[✓ A] 2^{20}

B $2^{20}-1$

C $2^{20}+1$

D 2^{21}

Solution

$${}^{21}C_0+{}^{21}C_1+{}^{21}C_2+.....+{}^{21}C_{10}=\frac{2^{21}}{2}=2^{20}.$$

#1613005

Topic: Locus and Its Equation

If $|z-1|=|z-i|$then locus of z is?

A A circle of radius 1

B A circle of radius $\frac{1}{2}$

[✓ C] A straight line passing through origin with slope 1

D A straight line passing through origin with slope -1

Solution

Let $z = x + iy$

Now given $|(x+iy)-1| = |(x+iy)-i|$

$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = y$

Hence (3) is correct.

#1613007

Topic: Probability

Three vertices are chosen from the vertices of a regular hexagon, then the probability that they will form an equilateral triangle is?

A $\frac{1}{5}$

[✓ B] $\frac{1}{10}$

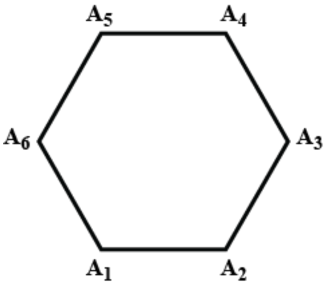
C $\frac{2}{15}$

D $\frac{2}{5}$

Solution

Choosing vertices of a regular hexagon alternate, here A_1, A_3, A_5 or A_2, A_4, A_6 will result in an equilateral triangle.

Hence required probability $= \frac{2}{{}^6C_3} = \frac{1}{10}$



#1613011

Topic: Variance and Standard Deviation

If expected value in n Bernoulli trials is 8 and variance is 4. If $P(x \leq 2) = \frac{k}{2^{16}}$ then value of k is?

A 1

[✓ B] 137

C 136

D 120

Solution

Let number of trials be n and probability of success =p, probability of failure =q

Given $np = 8, npq = 4$

$\Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 16$(as $p + q = 1$)

$p(x \leq 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2 2^{16}}{=} \frac{1 + 16 + 120}{2^{16}} = \frac{137}{2^{16}}$

Hence (2).

#1613014

Topic: First Principle of Differentiation

$e^y + xy = e$ then ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $y = 1$, is?

[✓ A] $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$

B $\left(\frac{1}{e},-\frac{1}{e^2}\right)$

C $\left(-\frac{1}{e},\frac{1}{e}\right)$

D $\left(\frac{1}{e},-\frac{1}{e}\right)$

Solution

$y=1\Rightarrow x=0$

$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$

$e\frac{dy}{dx}+1=0\Rightarrow\frac{dy}{dx}=-\frac{1}{e}$

$e^y\frac{d^2y}{dx^2}+e^y\left(\frac{dy}{dx}\right)^2+x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$

$x=0,y=1\Rightarrow$

$e\frac{d^2y}{dx^2}+e\left(-\frac{1}{e}\right)^2+0+2\left(-\frac{1}{e}\right)=0$

$\frac{d^2y}{dx^2}=\frac{1}{e^2}.$

#1613016

Topic: Direct Method

If α and β are the roots of the equation $375x^2-25x-2=0$ then the value of $\lim_{n\to\infty}\left(\sum_{r=1}^{n}\alpha^r+\sum_{r=1}^{n}\beta^r\right)$ the value of is?

[✓ A] $\frac{29}{248}$

B $\frac{17}{348}$

C $\frac{29}{358}$

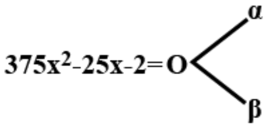
D $\frac{11}{348}$

Solution

Both roots lie in $(-1,1)$ hence sum of given series is finite

$\lim_{n\to\infty}\left(\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}\right)=\frac{\alpha(1-\beta)+\beta(1-\alpha)}{(1-\alpha)(1-\beta)}$

$=\frac{(\alpha+\beta)-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}=\frac{25-2(-2)}{375-25-2}=\frac{29}{348}.$



#1613018

Topic: Trigonometric Functions

$\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to?

[✓ A] $\frac{\pi}{2}-\cos^{-1}\left(\frac{63}{65}\right)$

B $\pi-\cos^{-1}\left(\frac{33}{65}\right)$

C $\frac{\pi}{2}-\sin^{-1}\left(\frac{33}{65}\right)$

D $\frac{\pi}{2}+\sin^{-1}\left(\frac{63}{65}\right)$

Solution

$\sin^{-1}\frac{12}{13}-\sin^{-1}\frac{3}{5}=\sin^{-1}\left(\frac{12}{13}\cdot\frac{4}{5}-\frac{3}{5}\cdot\frac{5}{13}\right)=\sin^{-1}\frac{33}{65}=\frac{\pi}{2}-\cos^{-1}\frac{33}{65}$

#1613028

Topic: Higher Order Derivatives

A ladder is 2m long has lower end on the ground and the other end in contact with a vertical wall. The lower end slips along the ground. If upper end is moving downward at th rate of 25 cm/sec and when upper end of ladder is 1m above the ground, then the rate at which lower end of the ladder is sliding on the ground is?

[✓ A] $-\dfrac{25}{\sqrt{3}}$

B $\dfrac{22}{\sqrt{3}}$

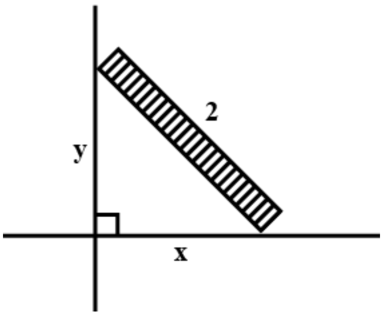
C $-\dfrac{22}{\sqrt{3}}$

D $\dfrac{25}{\sqrt{3}}$

Solution

$x^2 + y^2 = 4 \Rightarrow 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0 \Rightarrow \dfrac{dx}{dt} = -\dfrac{y}{x}\cdot\dfrac{dy}{dt}$

When upper end is 1m above the ground, $\dfrac{dx}{dt} = -\dfrac{1}{\sqrt{3}}\cdot 25 = -\dfrac{25}{\sqrt{3}}$cm/sec.



#1613036

Topic: Functions

Consider $f(x) = x\sqrt{kx - x^2}$ for $x \in [0, 3]$. Let m be the smallest value of k for which the function is increasing in the given interval and M be the largest value of $f(x)$ for that value of k. Then ordered pair (m, M) is?

A $(3, 3\sqrt{3})$

B $(5, 3\sqrt{3})$

C $(3, 5\sqrt{3})$

[✓ D] $(4, 3\sqrt{3})$

Solution

$f(x) = x\sqrt{kx - x^2}$

$f'(x) = \sqrt{kx - x^2} + \dfrac{(k-2x)x}{2\sqrt{kx - x^2}} = \dfrac{2(kx - x^2) + kx - 2x^2}{2\sqrt{kx - x^2}} = \dfrac{3kx - 4x^2}{2\sqrt{kx - x^2}} = \dfrac{x(3k - 4x)}{2\sqrt{kx - x^2}}$

for increasing function for $f'(x) \geq 0 \forall x \in [0, 3]$

$\Rightarrow kx - x^2 \geq 0, \forall x \in [0, 3]$and $x(3k - 4x) \geq 0, \forall x \in [0, 3]$

$\Rightarrow x(x - k) \leq 0, \forall x \in [0, 3]$and $(4x - 3k) \leq 0, \forall x \in [0, 3]$

$k \geq 3$and $k \geq 4$

$\Rightarrow k \geq 4$

$\Rightarrow m = 4$

maximum $(f(x))$ when $k = 4$is $3\sqrt{4 \times 3 - 3^2} = 3\sqrt{3} = M$

$9m, M) = (4, 3\sqrt{3})$

#1613050

Topic: Operations on Matrices

Let $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, where A is a symmetric matrix and B is a skew symmetric matrix, then $A \times B$is equal to?

A $\begin{bmatrix} 4 & 2 \\ 1 & 4 \end{bmatrix}$

B $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

C $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

D $\begin{bmatrix} -4 & 2 \\ 1 & -4 \end{bmatrix}$

Solution

$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = P$(say)

Now $A = \dfrac{P + P^T}{2}$ & $B = \dfrac{P - P^T}{2}$

So $A = \dfrac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}\right) = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$

$B = \dfrac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So $AB = \left(\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

#1613053

Topic: Equation of Hyperbola

In what ratio, the point of intersection of the common tangents to hyperbola $\dfrac{x^2}{1} - \dfrac{y^2}{8} = 1$ and parabola $y^2 = 12x$, divides the foci of the given hyperbola?

A $3:4$

B $3:2$

C $5:4$

D $5:3$

Solution

Let equation of common tangent is $y = mx + \dfrac{3}{m}$

$\therefore \left(\dfrac{3}{m}\right)^2 = 1.m^2 - 8$

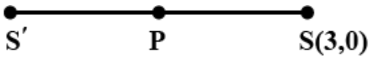
$\Rightarrow m^4 - 8m^2 - 9 = 0$

$\Rightarrow m^2 = 9$

$\Rightarrow m = \pm 3$

$\therefore$ equation of common tangents are $y = 3x + 1$ & $y = -3x + 1$

$\therefore \dfrac{PS}{PS'} = \dfrac{3 + \dfrac{1}{3}}{-\dfrac{1}{3} + 3} = \dfrac{5}{4}$.



#1613058

Topic: Introduction

If $y = \sin x \cdot \sin(x+2) - \sin^2(x+1)$ represents a straight line, then it passes through?

A I and II quadrant

B I, II and III quadrant

C I, III, IV quadrant

D III and IV quadrant

Solution

$y = \sin x \cdot \sin(x+2) - \sin^2(x+1) = \dfrac{1}{2}\{2\sin(x+2)\sin x - 2\sin^2(x+1)\}$

$= \dfrac{1}{2}\{\cos 2 - \cos(2x+2) + \cos(2x+2) - 1\} = -\sin^2 \cdot 1 < 0$

Hence the line passes through III & IV quadrant.

#1613067

Topic: Operations on Vector

Let $\vec{a} = \lambda\hat{i} + \hat{j}, \vec{b} = \lambda\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \lambda\hat{k}$ are co-terminous edges of parallelepiped then the value of λ for which the volume of parallelepiped is minimum, is?

[✓ A] $\frac{1}{\sqrt{3}}$

B $-\frac{1}{\sqrt{3}}$

C 0

D $\frac{1}{\sqrt{2}}$

Solution

$v = [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \lambda(\lambda^2 - 1) - 1 \cdot (0 - 1) = \lambda^3 - \lambda + 1$

Whose minimum value occur at $\lambda = \frac{1}{\sqrt{3}}$.

#1613073

Topic: Position of a Point w.r.t Ellipse

Let a curve satisfying the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$which passes through $(1, 1)$. If the curve also passes through $(k, 2)$, then value of k is?

A $\frac{1}{2} - \frac{1}{\sqrt{e}}$

B $\frac{3}{2} + \frac{1}{\sqrt{e}}$

[✓ C] $\frac{3}{2} - \frac{1}{\sqrt{e}}$

D $\frac{1}{2} + \frac{1}{\sqrt{e}}$

Solution

$y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

Integrating factor (I.F.)= $e^{-\frac{1}{y}}$

Now $x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3} dy$

Put $-\frac{1}{y} = y$

$x.e^t = \int e^t(-t)dt$

$\Rightarrow x.e^t = -(t.e^t - e^t) + c$

$\Rightarrow e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(1 + \frac{1}{y}\right) + c$

$\Rightarrow x = 1 + \frac{1}{y} + c.e^{\frac{1}{y}}$

it passes through point $(1, 1)$

$\therefore c = -\frac{1}{e}$

Equation of curve is

$x = 1 + \frac{1}{y} - e^{\frac{1}{y} - 1}$

It passes through $(k, 2)$

$\therefore k = 1 + \frac{1}{2} - e^{-\frac{1}{2}} = \frac{3}{2} - \frac{1}{\sqrt{e}}$

#1613081

Topic: Definite Integrals

If $\int_0^{f(x)} 4x^3 dx = g(x)(x - 2)$if $f(2) = 6$ and $f'(2) = \frac{1}{48}$ then find $\lim_{x \to 2} g(x)$.

[✓ A] 18

B 17

C 20

D 19

Solution

Answer: 1 or Bonus

$\int_0^{f(x)} 4x^3 dx = g(x) \cdot (x-2)$

$\Rightarrow g(x) = \frac{(f(x))^4}{x-2}$

$\therefore \lim\limits_{x \to 2} g(x) = \lim\limits_{x \to 2} \frac{(f(x))^4}{x-2} = \lim\limits_{x \to 2} \frac{4f^3(x) \cdot f'(x)}{1} = 4 \times 6^3 \times \frac{1}{48} = 18$

But $g(x) = \frac{\int_0^{f(x)} 4x^3 dx}{x-2}$ is not $\frac{0}{0}$ from as $f(2) = 6$.

Note: As per data received from the students we believe it will be Bonus.