#1613219

Topic: Definite Integrals

If $\int_{\alpha}^{2\alpha+1} \dfrac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\dfrac{9}{8}\right)$ then number of values of α is

- **A** 2 ☐
- **B** 3
- **C** 4
- **D** 6

Solution

Let $x+\alpha = t$

$dx = dt$

$$I = \int_{\alpha}^{2\alpha+1} \frac{dt}{t(t+1)} = \int_{\alpha}^{2\alpha+1}\left(\frac{1}{t} - \frac{1}{t+1}\right)dx = |\ln t - \ln(t+1)|_{2\alpha}^{2\alpha+1}$$

$$= \ln\left|\ln\left(\frac{t}{t+1}\right)\right|_{2\alpha}^{2\alpha+1} = \ln\left(\frac{2\alpha+1}{2\alpha+2}\right) - \ln\left(\frac{2\alpha}{2\alpha+1}\right) = \ln\left(\frac{2\alpha+1}{2\alpha+2}\times\frac{2\alpha+1}{2\alpha}\right) = \ln\left(\frac{9}{8}\right)$$

$\Rightarrow (2\alpha+1)^2 = 9\alpha$

$(2\alpha+2) = 4$

$4\left(4\alpha^2+1+4\alpha\right) = 18\alpha^2 + 18\alpha$

$2\alpha^2 + 2\alpha - 4 = 0$

$(\alpha+2)(\alpha-1) = 0$

$\alpha = 1 \quad or \quad \alpha = -2$

Number of values of α are 2

#1613224

Topic: Combination

A team of three persons with at least one boy and atleast one girl is to be formed from 5 boys and n girls. If the number of sum teams is 1750, then the value of n is

- **A** 24
- **B** 28
- **C** 27
- **D** 25 ☐

Solution

Given 5 boys, n girls

$(1B, 2G) + (2B, 1G)$

$${}^5C_1.{}^nC_2 + {}^5C_2.{}^nC_1 = 1750 \Rightarrow 5.\frac{n(n-1)}{2} + 10.n = 1750 \Rightarrow \frac{n(n-1)}{2} + 2n = 350 \Rightarrow n^2 - n + 4n = 700$$

$n^2 + 3n - 700 = 0 \Rightarrow (n+28)(n-25) = 0 \Rightarrow n = 25, -28$

#1613230

Topic: Profit and Loss

Two fair dice are thrown simultaneously. If both die show the same numbers, then the person wins Rs. 15. If the sum of numbers is 9, he wins Rs. 12. In all other cases, he loses Rs. 6. Then the expectation is

- **A** Rs. 2 gain
- **B** Rs. $1/2$ gain
- **C** Rs. 1.2 loss ☐
- **D** Rs. 2 loss

Solution

When two dice are thrown

Sample space $= \{(1,1),(2,2),.....(6,6)\}$contain total 36 elements number of cases when both dice

Expedation $= \frac{6}{36} \times 15 + \frac{4}{36} \times 12 - \frac{26}{36} \times 6$

$\frac{90+48-156}{36} = -\frac{1}{2}$

#1613231

Topic: Geometric Progression

If α, β, γ are non-constant terms in G.P and equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$has a common root then $(\gamma - \alpha), \beta$is

A $\alpha\beta$

B $\beta\gamma$

C $\gamma\alpha$

D 0

Solution

Let the common ratio of G.P is r Therefore $\beta = \alpha t, \alpha t^2$

Equation $$\alpha { x }^{ 2 }+2\alpha rx+\alpha { t }=0\quad

\Rightarrow { x }^{ 2 }+2rx+{ t }^{ 2 }=0....(i)$$

Given equation (i) and $x^2 + x - 1 = 0....(ii)$has a common root

$(i) - (ii) \Rightarrow (2e-1)x + (r^2+1) = 0 \Rightarrow x = \frac{-(r^2+1)}{2r-1}....(iii)$

Putting (iii) in equation (ii) $\Rightarrow (r^2+1)^2 - (r^2+1)(2r-1) - (2r^2-1)^2 = 0 \Rightarrow r^4 - 2r^3 - r^2 + 2r + 1 = 0....(iv)$

dividing equation (iv) by $r^2 \Rightarrow \left(r - \frac{1}{r}\right)^2 - 2\left(r - \frac{1}{r}\right) + 1 = 0 \Rightarrow \left(r - \frac{1}{r} - 1\right)^2 = 0 \Rightarrow \frac{r-1}{r} = 1....(v)$

$(\gamma - \alpha)\beta = (\alpha r^2 - \alpha) \times \alpha r = \alpha^2 (\alpha^2 - 1) r = \alpha^2 (r-1) = \alpha^2 r^2$

(using $(v) = \alpha \times \alpha t^2$

#1613232

Topic: Fundamental Integrals

If $\int \frac{\tan x - \tan\alpha}{\tan x + \tan\alpha} dx = f(x).\cos 2\alpha + g(x).\sin 2\alpha + c$ then $f(x)$ and $g(x)$ respectively are

A $x, \ln\sin(x+\alpha)$

B $\cos 2x, \ln\sin(x+2\alpha)$

C $\cos 2x, -\ln\sin(x+\alpha)$

D $x, -\ln\sin(x+\alpha)$

#1613233

Topic: Truth Tables

Negation of the statement $p \to (p\vee \sim q)$ is

A $p \vee q$

B $p \wedge q$

C f

D t

Solution

$p \to (p\vee \sim q)$

$-(p \to (p\vee \sim q))$

$\because -(p \to q) = p\wedge \sim q$

$= p\wedge \sim (p\vee \sim q) = p \wedge (\sim p\wedge \sim q) = (p\wedge \sim p) \wedge (p \wedge q) = f \wedge (p \wedge q) = f$

#1613234

Topic: Arithmetic Progression

If $a_1, a_2, ... a_n$ are in A.P with $a_1 + a_7 + a_{16} = 40$ Then the value of $a_1 + a_2 + ... a_{15}$ is

A 260

B 240

C 200

D 160

Solution

$a_1 + a_7 + a_{16} = 40 \Rightarrow a_1 + (a_1 + 6d) + (a_1 + 15d) = 40 \Rightarrow 3a_1 + 21d = 40 \Rightarrow a_1 + 7d = \frac{40}{3}$

$a_1 + a_2 + ... + a_{15} \Rightarrow \frac{15}{2}(a_1 + a_{15}) \Rightarrow \frac{15}{2}[a_1 + a_1 + 14d] = 15(a_1 + 7d) = 15 \times \frac{40}{3} = 200 \qquad 15$

(3) option is correct

#1613235

Topic: Construction of Triangles

Then angle of elevation of a top of a tower (point B) from a point A on ground at horizontal distance d from foot of the tower is 45^o. Angle of elevation from a point C $30m$ vertically above point A is 30^o. Then the value of d is

A $15\left(\sqrt{3}+1\right)$

B $15\left(3+\sqrt{3}\right)$

C $30\left(\sqrt{3}-1\right)$

D $30\left(3+\sqrt{3}\right)$

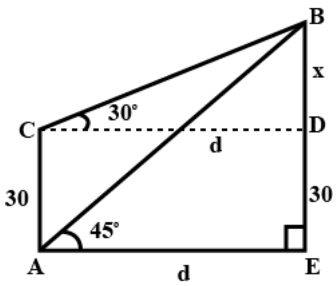
Solution

$\frac{x}{d} = \tan 30^o; \frac{x}{d} = \frac{1}{\sqrt{3}}; d = \sqrt{3}x$

$\frac{x+30}{d} = \tan 45^o \Rightarrow d = x + 30$

$d = \frac{d}{\sqrt{3}} + 30 \Rightarrow \left(1 - \frac{1}{\sqrt{3}}\right) d = 30 \Rightarrow d = \frac{30\sqrt{3}}{\sqrt{3}-1} \Rightarrow d = \frac{30\sqrt{3}(\sqrt{3}+1)}{2} = 15\sqrt{3}(\sqrt{3}+1)$

$d = 15\left(3+\sqrt{3}\right)$



#1613236

Topic: Types of Solution of Differential Equation

Solution of the differential equation of $(y^2 - x^3)dx - xydy = 0$ is

A $y^2 + 2x^3 + cx^2 = 0$

B $y^2 - 2x^3 + cx^2 = 0$

C $y^2 + 2x^3 - cx^2 = 0$

D $y^2 + 2x^3 + cx^2 = 0$

Solution

$(y^2-x^3)dx - xydy = 0$

$\Rightarrow y(ydx - xdy) = x^3 dx \Rightarrow \frac{y}{x}\left(\frac{ydx - xdy}{x^2}\right) = dx \Rightarrow \frac{y}{x} d\left(\frac{y}{x}\right) = dx \Rightarrow -\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + k$

$\Rightarrow -y^2 = 2x^3 + 2x^2 k \Rightarrow y^2 + 2x^3 + cx^2 = 0$

#1613240

Topic: Area of Bounded Regions

If the area enclosed by the curves $y^2 = 4\lambda x$ and $y = \lambda x$ is $\frac{1}{9}$ square units then value of λ is

A 24

B 37

C 48

D 38

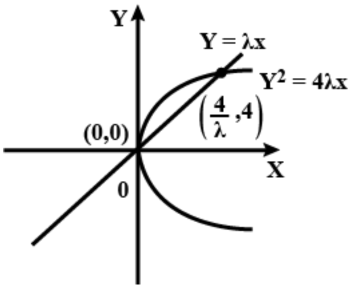
Solution

$y^2 = 4\lambda x \quad ; y = \lambda x$

If $\lambda > 0$ then

Hence $\int_0^{4/\lambda} \left(2\sqrt{\lambda}\sqrt{x} - \lambda x\right) dx = \frac{1}{9}$

$\left(\frac{2\sqrt{\lambda}x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\right)^{\frac{4}{\lambda}} = \frac{1}{9} \Rightarrow \frac{4}{3}\sqrt{\lambda}\frac{8}{\lambda^{3/2}} - \lambda\frac{8}{\lambda^2} = \frac{1}{9} \Rightarrow \frac{32}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$



#1613242

Topic: Special Series

The value of $1^2.\,{}^{20}C_1 + 2^2.\,{}^{20}C_2 + 3^2.\,{}^{20}C_3 + \ldots\ldots(20)^2.\,{}^{20}C_{20}$ is

A 210×2^{17}

B 420×2^{17}

C 420×2^{87}

D 210×2^{87}

Solution

$S = 1^2.\,{}^{20}C_1 + 2^2.\,{}^{20}C_2 + 3^2.\,{}^{20}C_3 + \ldots\ldots(20)^2.\,{}^{20}C_{20} = \sum_{r=1}^{20} r^2.\,{}^{20}C_r$

$= \sum_{r=1}^{20} r(r.\,{}^{20}C_r) = 20\sum_{r=1}^{20} r^{19}.\,{}^{19}C_{r-1} = 20\sum_{r=1}^{20} (r-1+1).\,{}^{19}C_{r-1} = 20\sum_{r=1}^{20} (r-1).\,{}^{19}C_{r-1} + 20\sum_{r=1}^{20} r^2.\,{}^{19}C_{r-1} = 20 \times 19\sum_{r=2}^{20} {}^{18}C_{r-1} + 20 \times 2^{19}$

${}^{19} = 20 \times 19 \times 2^{18} + 20 \times 2^{19} = 20 \times 2^{18}(19+2) = 20 \times 21 \times 2^{18} = 420 \times 2^{18}$

(3) option is correct

#1613248

Topic: Maths

If $\frac{2z-n}{2z+n} = 2i - 1, n \in N$ and $Im(z) = 10$, then

A $n = 20, Re(z) = 10$

B $n = 20, Re(z) = -10$

C $n = 40, Re(z) = 10$

D $n = 40, Re(z) = -10$

Solution

Let $Re(z) = x$,then

$\dfrac{2(x+10i)-n}{2(x+10i)+n} = 2i - 1 \Rightarrow (2x-n)+20i = -(2x+n) - 40 - 20i + 2ni$

$\Rightarrow 2x - n = 2x - n - 40; 20 = -20 + 2n \Rightarrow x = -10; n = 20$

(b) is correct option

#1613250

Topic: Differentiation by Substitution

The differentiation of $\tan^{-1}\left(\dfrac{\tan x+1}{\tan x-1}\right)$ where $x \in \left(0, \dfrac{\pi}{4}\right)$ with respect to $\dfrac{x}{2}$ is

A 2

B 1

C $\dfrac{2}{3}$

D -2

Solution

$y = \tan^{-1}\left(\dfrac{\tan x+1}{\tan x-1}\right) = -\tan^{-1}\left(\dfrac{\tan\dfrac{\pi}{4}\tan x}{1-\tan\dfrac{\pi}{4}\tan x}\right) = -\tan^{-1}\left(\tan\left(x+\dfrac{\pi}{4}\right)\right) = -\left(x+\dfrac{\pi}{4}\right)$

$\Rightarrow \dfrac{dy}{dx} = -1$

Now if differentiation of $\dfrac{x}{2}$ w.r.t x is $\dfrac{1}{2}$

differentiation of y w.r.t $\dfrac{x}{2}$ is $\dfrac{-1}{\frac{1}{2}} = -2$

#1613261

Topic: Probability

There are 50 questions in of an exam. A student's probability of getting an answer correct is $\dfrac{4}{5}$. Then the probability that he is unable to correctly answer less than 2 question

A $\dfrac{201}{5}(415)^{50}$

B $\dfrac{201}{5}\left(\dfrac{1}{5}\right)^{50}$

C $\dfrac{201}{5}\left(\dfrac{1}{5}\right)^{50}$

D $\dfrac{201}{5}\left(\dfrac{4}{5}\right)^{50}$

Solution

There are 50 questions in an exam

Probability of each question to be correct $= p = \frac{4}{5}$

Probability of each question is incorrect $= q = \frac{1}{5}$

Let $X =$ number of correct question in 50 questions

Hence required probability $= p(X=0) + p(X=1) == p(X=0) + p(X=1) = \left(\frac{1}{5}\right)^{50} + {}^{50}C_1 \left(\frac{4}{5}\right)^{1} \left(\frac{1}{5}\right)^{49} \quad = \left(\frac{1}{5}\right)^{50} (1+200) = 201\left(\frac{1}{5}\right)^{50}$

#1613274

Topic: Position of a Point w.r.t Ellipse

An ellipse with foci $(0, \pm 2)$ has length of minor axis as 4 units. Then the ellipse will pass through the point

A $(2, \sqrt{2})$

B $(\sqrt{2}, 2)$

C $(2, 2\sqrt{2})$

D $(2\sqrt{2}, 2)$

Solution

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$ is the equation of ellipse, foci $(0, \pm 2)$

(be $= 2$)

Given: $2a = 4 \Rightarrow a = 2$

$e^2 = 1 - \frac{a^2}{b^2} \Rightarrow b^2 e^2 = b^2 - a^2$

$4 = b^2 - 4 \Rightarrow b^2 = 8$

$\therefore$ equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{8} = 1$

It passes through $(\sqrt{2}, 2)$

#1613280

Topic: Tangent

The equation of common tangent to the curves $y^2 = 16x$ and $xy = -4$ is

A $y = x + 4$

B $2x - y + 8 = 0$

C $x + y = 4$

D $2x + y + 4 = 0$

Solution

Let equation of tangent to parabola $y^2 = 16x$ is $y = mx + \frac{4}{m} \dots. (1)$

It is tangent to $xy = -4 \dots (2)$ solving (1) and (2) we get

$x\left(mx + \frac{4}{m}\right) + 4 = 0 \Rightarrow mx^2 + \frac{4}{m}x + 4 = 0$

for tangent $D = 0 \Rightarrow \frac{16}{m^2} - 16m = 0 \Rightarrow m^3 = 1 \Rightarrow m = 1$

Putting $m = 1$ in equation (1) Common tangnet is $y = x + 4$

#1613292

Topic: Functions

If $[-\sin\theta]\, x + [\cos\theta]\, y = 0 \quad [\cot\theta]\, x + y = 0$ where [] denotes greatest integer function. Then which the following is correct.

A Infinite solutions in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and a unique solution in $\left(\pi, \frac{7\pi}{6}\right)$

B Unique solutions in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and a infinite solution in $\left(\pi, \frac{7\pi}{6}\right)$

C Unique solutions in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and a unique solution in $\left(\pi, \frac{7\pi}{6}\right)$

D Infinite solutions in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and a infinite solution in $\left(\pi, \frac{7\pi}{6}\right)$

Solution

If $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow \sin\theta \in \left(\frac{\sqrt{3}}{2}, 1\right) \Rightarrow -\sin\theta \in \left(-1, \frac{-\sqrt{3}}{2}\right)$

$\cos\theta \in \left(-\frac{1}{2}, 0\right)$; $\cot\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

If $\theta \in \left(\pi. \frac{7\pi}{6}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{2}, 0\right) \Rightarrow -\sin\theta \in \left(0, \frac{1}{2}\right)$

$\cos\theta \in \left(-1, \frac{-\sqrt{3}}{2}\right)$; $\quad \cos\theta \in (\sqrt{3}, \infty)$

Hence in $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow (-\sin\theta) = -1; (\cos\theta) = -1; (\cot\theta) = -1$

Hence $-x - y = 0$and $-x + y = 0$has unique solution

in $\theta \in \left(\pi. \frac{7\pi}{6}\right) \Rightarrow (-\sin\theta) = 0; (\cos\theta) = -1; (\cot\theta) = 1, 2, 3, \ldots$ Hence $0, x - y = 0 \Rightarrow y = 0$

$(\cot\theta)\, x + y = 0 \Rightarrow 1.x + y = 0 \quad or 2x + y = 0$each line well cut $y = 0$at exactly one point

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#1613299

Topic: Limits

Value of $\lim_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{x - \sin^2 x + 1}}$ is

A 2

B 1

C 6

D -2

Solution

$$\lim_{x\to 0} \frac{(x + 2\sin x)\left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{x + \cos^2 x}\right)}{(x^2 + 2\sin x + 1) - (x + \cos^2 x)} = \lim_{x\to 0} \frac{(x + 2\sin x)\left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{x + \cos^2 x}\right)}{x^2 + 2\sin x + 1 + 1 - x + \sin^2 x - 1}$$

$$\lim_{x\to 0} \frac{(x + 2\sin x)\left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{x + \cos^2 x}\right)}{x^2 - x + 2\sin x + \sin^2 x} = \lim_{x\to 0} \frac{\left(x + 2\frac{\sin x}{x}\right)\left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{x + \cos^2 x}\right)}{x^2 + 2\frac{\sin x}{x} + \sin x \times \left(\frac{\sin x}{x}\right)} = \frac{(1+2)(1+1)}{0 - 1 + 2 + 0} = 6$$

#1613302

Topic: Types of Sets

If $A \cap B \subseteq C$ and $A \cap B \neq \phi$ Then which of the following is incorrect

A $(A \cup B) \cap C \neq \phi$

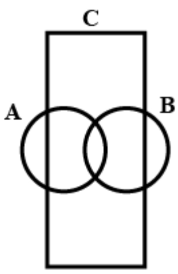
B $B \cap C = \phi$

C $A \cap C \neq \phi$

D If $(A - C) \subseteq C$ then $A \subseteq C$

Solution

From the figure, it is clear that option (2) is correct



#1613305

Topic: Limits

Lte $f(x) = 5 - (x - 2)$

$g(x) = (x+1)(x+3)$ If maximum value of $f(x)$ is α

and minimum value of $f(x)$ is β then $\lim_{x \to (\alpha - \beta)} \dfrac{(x-3)\left(x^2 - 5x + 6\right)}{(x-1)\left(x^2 - 6x + 8\right)}$ is

- **[A]** $-\dfrac{1}{2}$
- B $-\dfrac{1}{2}$
- C $\dfrac{3}{2}$
- D $-\dfrac{3}{2}$

Solution

maximum value of $f(x)$ is $\alpha = 5$

minimum value of $f(x)$ is $\beta = 3$

$$\lim_{x \to 2} \frac{(x-3)(x-2)(x-3)}{(x-1)(x-2)(x-4)} = \lim_{x \to 2} \frac{(x-3)(x-3)}{(x-1)(x-4)} = -\frac{1}{2}$$

#1613310

Topic: Properties of Matrices

If $\begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4\cos 6\theta \end{vmatrix} = 0$ and $\theta \in \left(0, \dfrac{\pi}{2}\right)$, then value of θ is

- A $\dfrac{7\pi}{36}$
- B $\dfrac{7\pi}{24}$
- **[C]** $\dfrac{\pi}{9}$
- D $\dfrac{\pi}{4}$

Solution

$$\begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1 + 4\cos 6\theta \end{vmatrix} = 0$$

$C_2 \to C_2 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1 + 4\cos 6\theta \end{vmatrix} = 0$$

$$\Rightarrow 1 + 4\cos 6\theta + 1 = 0 \Rightarrow 2\cos 6\theta = -1 \Rightarrow \cos 6\theta = -\frac{1}{2} = \cos\frac{2\pi}{3} \Rightarrow 6\theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{3} \pm \frac{\pi}{9}, n \in 1 \Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$$

#1613312

Topic: Position of a Point/Line w.r.t Circle

A circle touches x-axis at point $(3, 0)$. If it makes an intercept of 8 units on y-axis, then the circle passes through which point

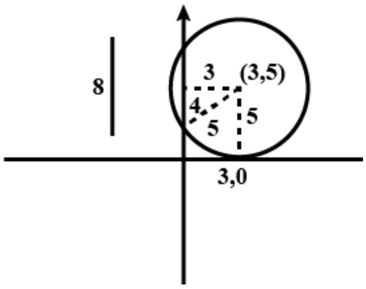
- A $(3, 1)$
- B $(5, 2)$
- C $(10, 3)$

D	$(3, 10)$

Solution

Equation of circle is $(x-3)^2+(y-5)^2=5^2$

Hence, $(3, 10)$ will satisfy equation



#1613317
Topic: Angles and Properties

A line is at distance of 4 units from origin and having both intercepts positive. If the perpendicular from the origin to this line makes an angle of 60^o with the line $x+y=0$Then the equation of the line is

A $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}+1\right)y=y=8\sqrt{2}$

B $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}+1\right)y=y=8\sqrt{2}$

C $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

D $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

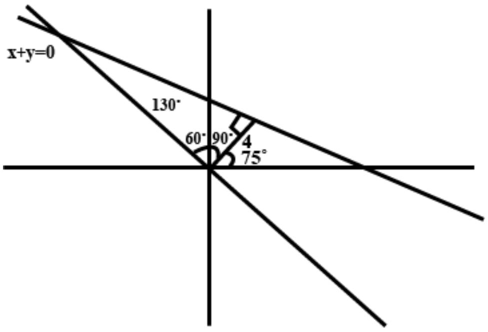
Solution

Hence equation of $lineis$x\cos { \theta } +y\sin { \theta } =p$$

$$x\cos { \left({ 75 }^{ o } \right) } +y\sin { \left({ 75 }^{ o } \right) } =4

$$x\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+y\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)=4$$

$$x\left(\sqrt{3}-1\right)+y\left(\sqrt{3}+1\right)=8\sqrt{2}$$



#1613320
Topic: Solving Quadratic Equation

If the equation $\cos 2x+\alpha\sin x=2\alpha-7$ has a solution. Then range of α is

A $[2, 6]$

B $[-2, 6]$

C $[-6, -2]$

D None of these

Solution

$$1-2\sin^2 x+\alpha\sin x=2\alpha-7\Rightarrow 2\sin^2 x-\alpha\sin x+(2\alpha-8)=0$$

$\sin x=\dfrac{\alpha\pm\sqrt{\alpha^2-8\left(2\alpha-8\right)}}{4}=\dfrac{\alpha\pm\sqrt{\alpha^2-16\alpha+64}}{4}=\dfrac{\alpha\pm\sqrt{\alpha-8}}{4}=\dfrac{2\alpha-8}{4}, 2=\dfrac{\alpha-4}{2}, 2$(Rejected), for solution to be exist

$$-1\leq\frac{\alpha-4}{2}\leq 1\Rightarrow -2\leq\alpha-4\leq 2\Rightarrow 2\leq\alpha\leq 6\Rightarrow\alpha\in[2, 6]$$

#1613331
Topic: Position of Point w.r.t Parabola

If a line $x - y = 3$intersects the parabola $y = (x-2)^2 - 1$at A and B and tangents at A and B meet again at point C. Then coordinates of point C is

A $\left(-\frac{5}{2}, -1\right)$

B $\left(-\frac{5}{2}, 1\right)$

C $\left(\frac{5}{2}, -1\right)$

D $\left(\frac{5}{2}, 1\right)$

#1613338
Topic: Triangles

A triangle having a vertex as $(1, 2)$ has mid point of sides passing from this vertex as $(-1, 1)$ and $(2, 3)$. Then the centroid of the triangle is

A $\left(\frac{1}{3}, 2\right)$

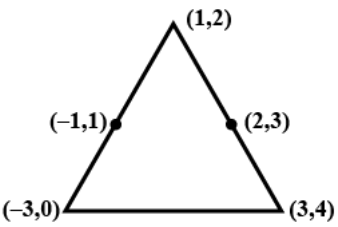
B $\left(2, \frac{1}{3}\right)$

C $(1, 1)$

D $\left(\frac{1}{3}, 4\right)$

Solution

centroid $= \left(\frac{1+3-3}{3}, \frac{2+0+4}{3}\right) = \left(\frac{1}{3}, 2\right)$



#1613341
Topic: Maths

The term independent of x in $\left(\frac{1}{60} - \frac{x^8}{81}\right)\left(2x^2 - \frac{3}{x^2}\right)^6$ is

A -36

B 36

C 72

D 108

Solution

The term independent of x

$$= \frac{1}{60} \times 2^3 \times (-3)^3 \times {}^6C_3 + \left(-\frac{1}{81}\right)(2)(-3) \times {}^6C_1 = -72 + 36 = -36$$

#1613353
Topic: Plane

The point lying on angle bisector of the planes $x + 2y + 2z - 6 = 0$and $2x - y + 4 = 0$is

A $(2, 4, 0)$

B $(-1, 3, 2)$

C $(-1, -3, 2)$

D $(-2, 4, 0)$