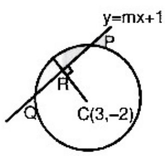
JEE Advanced Answer Key 2019
Maths Paper 1

1. A line $y = mx + 1$ meets the circle $(x - 3)^2 + (y + 2)^2 = 25$ at points P and Q. If mid point of PQ has abscissa of $-\frac{3}{5}$, then value of m satisfies

(A) $6 \le m < 8$ (B) $2 \le m < 4$ (C) $-3 \le m < -1$ (D) $4 \le m < 6$

Ans. **(B)**

Sol.



For point R, $x = -\frac{3}{5} \Rightarrow y = 1 - \frac{3m}{5}$ $\quad R\left(-\frac{3}{5}, 1-\frac{3m}{5}\right)$

slope of CR $= \dfrac{1-\frac{3m}{5}+2}{-\frac{3}{5}-3} = -\frac{1}{m} \Rightarrow \quad \dfrac{15-3m}{-3-15} = -\frac{1}{m}$

$15m - 3m^2 = 18$

$m^2 - 5m + 6 = 0$

$m = 2, 3$

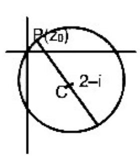
$2 \le m \le 4$

2. If z is a complex number belonging to the set $S = \{z : |z - 2 + i| \ge \sqrt{5}\}$ and $z_0 \in S$ such that $\frac{1}{|z_0 - 1|}$ is maximum. Then $\arg\left(\dfrac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

Ans. **(C)**

Sol. E



$|z - (2 - i)| \ge \sqrt{5}$

For $|z_0 - 1|$ to be minimum, $z_0 = x_0 + iy_0$ is at point P as shown in figure

$\arg\left(\dfrac{4-(z_0+\bar{z}_0)}{z_0-\bar{z}_0+2i}\right) = \arg\left(\dfrac{4-2x}{2iy+2i}\right) = \arg\left(\dfrac{-i(2-x)}{y+2}\right) = \arg(-i\lambda) = -\frac{\pi}{2} \quad (\because \lambda > 0)$

3. Area bounded the points (x, y) in cartesian plane satisfying $xy \le 8$ and $1 \le y \le x^2$ will be

(A) $16\ell n2 - \frac{14}{3}$ (B) $8\ell n2 - \frac{7}{3}$ (C) $8\ell n2 - \frac{14}{3}$ (D) $16\ell n2 - 6$

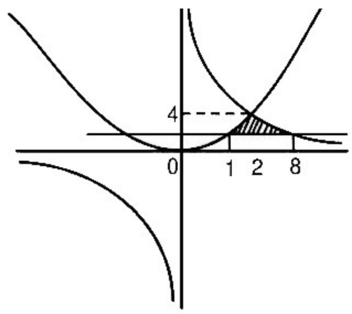
Ans. **(A)**

Sol. $xy \le 8$

$1 \le y \le x^2$

$x^2 . x = 8$

$x = 2$



Required Area = $\int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dy = \left[8\ell n\, y - \frac{y^{3/2}}{3/2}\right]_1^4 = 8\ \ell n4 - \frac{2}{3}\ .\ 8 - 0 + \frac{2}{3} = 16\ell n2 - \frac{14}{3}$

4. $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha\ (\theta)$ and $\beta = \beta\ (\theta)$ are real numbers and I is an identity matrix of 2×2.

If α^* = Min of set $\{\alpha(\theta) : \theta \in [0,2\pi)\}$

And β^* =Min of set $\{\beta(\theta) : \theta \in [0,2\pi)\}$

Then value of $\alpha^* + \beta^*$ is

$M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

(A) $\frac{-37}{16}$ (B) $\frac{-17}{16}$ (C) $\frac{-31}{16}$ (D) $\frac{-29}{16}$

Ans. **(D)**

Sol. $m = \sin^4\theta\ .\ \cos^4\theta + (1 + \sin^2\theta)(1 + \cos^2\theta)$

$2 + \sin^4\cos^4\theta + \sin^2\theta\cos^2\theta$

$\begin{bmatrix} \sin^4\theta & -(1+\sin^2\theta) \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \beta = \frac{1}{|m|} \begin{bmatrix} \cos^4\theta & 1+\sin^2\theta \\ -1-\cos^2\theta & \sin^4\theta \end{bmatrix}$

$\sin^4\theta = \frac{\alpha+\beta}{|m|} \cos^4\theta\ , -1 - \sin^2\theta = \frac{\beta}{|m|}(1 + \sin^2\theta)$

$\beta = -|m|$

$\beta = -[\sin^4\theta\cos^4\theta + \sin^2\theta\cos^2\theta + 2] = -[t^2 + t + 2] \Rightarrow \quad \beta_{min} = -\frac{37}{16}$

$\alpha = \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - \frac{1}{2}(\sin^2 2\theta) \quad \Rightarrow \quad \min\alpha = \frac{1}{2}$

$\alpha + \beta = -\frac{37}{16} + \frac{1}{2} = -\frac{37}{16} + \frac{8}{16} = -\frac{29}{16}$

5. If $a_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta}$ where α and β are roots of equation $x^2 - x - 1 = 0$ and $b_n = a_{n+1} + a_{n-1}$. Then

(A) $b_n = \alpha^n + \beta^n$ (B) $\sum_{n=1} \dfrac{b_n}{10^n} = \dfrac{8}{89}$ (C) $\sum_{n=1} \dfrac{a_n}{10^n} = \dfrac{10}{89}$ (D) $a_1 + a_2 + \ldots . a_n = a_{n+2} - 1$

Ans. **(ACD)**

Sol. (A) $b_n = a_{n+1} + a_{n-1} = \dfrac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} + \dfrac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} = \dfrac{\alpha^{n-1}(\alpha^2+1) - \beta^{n-1}(\beta^2+1)}{\alpha - \beta}$

$$= \frac{\alpha^{n-1}(\alpha+2) - \beta^{n-1}(\beta+2)}{\alpha-\beta} = \frac{\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right) - \beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)}{\alpha-\beta}$$

$$= \frac{\sqrt{5}\alpha^{n-1}\left(\frac{\sqrt{5}+1}{2}\right) - \sqrt{5}\beta^{n-1}\left(\frac{\sqrt{5}-1}{2}\right)}{\alpha-\beta} = \frac{\sqrt{5}(\alpha^n+\beta^n)}{\alpha-\beta} = \alpha^n + \beta^n \qquad \because \quad \alpha - \beta = \sqrt{5}$$

(B) $$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum\left(\frac{\alpha}{10}\right)^n + \sum\left(\frac{\beta}{10}\right)^n = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{\alpha}{10-\alpha} + \frac{\beta}{10-\beta}$$

$$= \frac{10(\alpha+\beta) - 2\alpha\beta}{100 - 10(\alpha+\beta) + \alpha\beta} = \frac{10+2}{89} = \frac{12}{89}$$

(C) $$\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \sum \frac{\alpha^n - \beta^n}{(\alpha-\beta)10^n} = = \frac{1}{\alpha-\beta}\left(\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right) \frac{1}{\alpha-\beta}\left(\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}\right)$$

$$= \frac{1}{\alpha-\beta} \cdot \frac{(10(\alpha-\beta) - \alpha\beta + \alpha\beta)}{100 - 10(\alpha+\beta) + \alpha\beta} = \frac{10}{89}$$ Option (C) is correct.

(D) $$a_1 + a_2 + \ldots . a_n = \Sigma a_i = \frac{\Sigma\alpha^i - \Sigma\beta^i}{\alpha-\beta} = \frac{\frac{\alpha(1-\alpha^n)}{(1-\alpha)} - \frac{\beta(1-\beta^n)}{(1-\beta)}}{\alpha-\beta}$$

$$= \frac{(\alpha+1)(1-\alpha^n) - (\beta+1)(1-\beta^n)}{(1-\alpha)(1-\beta)(\alpha-\beta)} = \frac{\alpha^2 - \alpha^{n+2} - \beta^2 + \beta^{n+2}}{(1-\alpha)(1-\beta)(\alpha-\beta)} = \frac{\sqrt{5} + \beta^{n+2} - \alpha^{n+2}}{\beta - \alpha} = -1 + a_{n+2}$$

6. If a matrix M is given by $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and if $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then

(A) $\text{adj}(M^{-1}) + (\text{adj}M)^{-1} = -M$ (B) $|\text{adj}(M^2)| = 81$

(C) $\alpha + 2\beta + 3\gamma = 2$ (D) $\beta + 2\gamma = 3$

Ans. **(AC)**

Sol. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow \beta + 2\gamma = 1$

$\alpha + 2\beta + 3\gamma = 2$

$3\alpha + \beta + \gamma = 3$

$\Rightarrow \alpha = 1, \beta = -1, \gamma = 1$

$|M| = -2$

$|\text{adj}M^2| = |M^2|^2 = |M|^4 = 16$

$\text{adj}(M^{-1}) = |M^{-1}|\, M = \dfrac{-M}{2}$

$(\text{adj}M)^{-1} = \text{adj}(M^{-1}) = -\dfrac{M}{2}$

7. There are three bags B_1, B_2, B_3. B_1 contains 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities 3/10, 3/10 and 4/10 respectively of being chosen. A bag is selected at random and a ball is randomly chosen from the bag. Then which of the following options is/are correct?

(A) Probability that the chosen ball is green equals $\frac{39}{80}$

(B) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(C) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{4}{13}$

(D) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{3}{10}$

Ans. (ABC)

Sol.

	Bag_1	Bag_2	Bag_3
Red Balls	5	3	5
Green Balls	5	5	3
Total	10	8	8

(A) $P(\text{Ball is Green}) = P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)$

$$= \frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}=\frac{39}{80}$$

(B) $P(\text{Ball chosen is Green/ Ball is from } 3^{rd} \text{ Bag}) = \frac{3}{8}$

(C,D) $P(\text{Ball is from } 3^{rd} \text{ Bag / Ball chosen is Green})$

$$= \frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1)+P(B_2)P(G/B_2)+P(B_3)P(G/B_3)}$$

$P(B_1) = \frac{3}{10}$

$P(B_2) = \frac{3}{10}$

$P(B_3) = \frac{4}{10}$

$$= \frac{\frac{4}{10}\times\frac{3}{8}}{\frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}} = \frac{4}{13}$$

8. Let L_1 and L_2 denote the lines $\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}),\ \lambda \in R$ and $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}),\ \mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ?

(A) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$

(B) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$

(C) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$

(D) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$

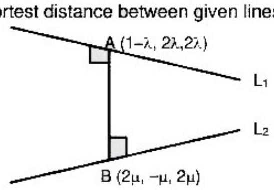
Ans. **(B,C,D)**

Sol. Both given lines are skew lines.

So direction ratios of any line perpendicular to these lines are $6\hat{i} + 6\hat{j} - 3\hat{k}$

<2, 2, −1>

Points at shortest distance between given lines are



$\overrightarrow{AB} \perp$ line L_1

$\overrightarrow{AB} \perp$ line L_2

So $A\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$

Now equation of required line $\vec{r} = \left(\frac{8}{9}\hat{i} + \frac{2}{9}\hat{j} + \frac{2}{9}\hat{k}\right) + \alpha(2\hat{i} + 2\hat{j} - \hat{k})$

Now by option B, C, D are correct.

9. Equation of ellipse E_1 is $\frac{x^2}{9} + \frac{y^2}{4} = 1$. A rectangle R_1, whose sides are parallel to co-ordinate axes is inscribed in E_1 such that its area is maximum. Now E_n is an ellipse inside R_{n-1} such that its axes is along co-ordinate axes and has maximum possible area $\forall\ n \ge 2,\ n \in N$. further R_n is a rectangle whose sides are parallel to co-ordinate axes and is inscribed in E_{n-1}. having maximum area $\forall\ n \ge 2,\ n \in N$.

(A) $\sum_{n=1}^{m}$ area of rectangle $(R_n) < 24\ \forall\ m \in N$

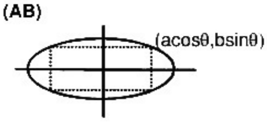
(B) Length of latus rectum of $E_9 = \frac{1}{6}$

(C) Distance between focus and centre of $E_9 = \frac{\sqrt{5}}{32}$

(D) The eccentricities of E_{18} and E_{19} are not equal

Ans. **(AB)**

Sol.



Area Max when $\theta = 45°$

	a	b
E_1	3	2
E_2	$\frac{3}{\sqrt{2}}$	$\frac{2}{\sqrt{2}}$
E_3	$\frac{3}{(\sqrt{2})^2}$	$\frac{2}{(\sqrt{2})^2}$
$\vdots$	$\vdots$	$\vdots$
E_9	$\frac{3}{(\sqrt{2})^8}$	$\frac{2}{(\sqrt{2})^8}$

(A) $E_1 + E_2 + \ldots\ldots + E_m$

when $m \to \infty$ $\frac{2ab}{1-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}} = 4ab = 4.3.2 = 24$

(B) Length of LR is ellipse $= \frac{2b^2}{a} = 2.\frac{4.2^4}{2^8.3} = \frac{1}{6}$

(C) distance between focus and center of ellipse $= a_9\, e_9 = \frac{3}{2^4}.\frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{16}$

10. In a non right angled triangle ΔPQR, let p,q,r denote the lengths of the sides opposite to the angle P,Q,R respectively. The median from R meets the side PQ at S. the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$ and the radius of the circumcircle of the ΔPQR equals to 1, then which of the following options is/are correct ?

(A) length of RS $= \frac{\sqrt{7}}{2}$

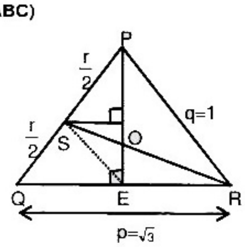
(B) length of OE $= \frac{1}{6}$

(C) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2-\sqrt{3})$

(D) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

Ans. **(ABC)**

Sol.



$$\frac{p}{\sin P} = \frac{q}{\sin Q} = 2(1) \qquad \Rightarrow \qquad \sin P = \frac{\sqrt{3}}{2}, \ \sin Q = \frac{1}{2}$$

$\Rightarrow \qquad \angle P = 60°$ or $120°$ and $\angle Q = 30°$ or $150°$

because $\angle P + \angle Q$ must be less than 180° but not equal to 90°

$\angle P = 120°$ and $\angle Q = 30°$ and $\angle R = 30° \qquad \frac{r}{\sin R} = 2 \Rightarrow r = 1$

Now length of median $RS = \frac{1}{2}\sqrt{2p^2 + 2q^2 - r^2} = \frac{1}{2}\sqrt{6+2-1} = \frac{\sqrt{7}}{2} \Rightarrow$ option (A) is correct

$$\text{Inradius} = \frac{2\Delta}{p+q+r} = \frac{\frac{2pqr}{4\times(1)}}{p+q+r} = \frac{1}{2}\left(\frac{1\times1\times\sqrt{3}}{1+1+\sqrt{3}}\right) = \frac{\sqrt{3}}{2}\left(\frac{2-\sqrt{3}}{1}\right) \Rightarrow \text{option (C) is correct}$$

$$\Rightarrow \qquad \frac{1}{2}\times\sqrt{3}\times PE = \frac{pqr}{4(1)} \ \text{(equal area of } \Delta) \qquad \Rightarrow \qquad PE = \frac{1\times1\times\sqrt{3}}{4}\times\frac{2}{\sqrt{3}} = \frac{1}{2}$$

$$\Rightarrow \qquad OE = \frac{2(\text{Area of } \Delta OQR)}{QR} = \frac{2\times\frac{1}{3}\left(\frac{1}{2}.1.\sqrt{3}\sin 30°\right)}{\sqrt{3}} = \frac{1}{6}$$

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11. Let T denote a curve y = f(x) which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to T at a point P intersect the y-axis at Y_P and PY_p has length 1 for each point P on T. Then which of the following option may be correct?

(A) $y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

(B) $xy' - \sqrt{1-x^2} = 0$

(C) $y = -\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(D) $xy' + \sqrt{1-x^2} = 0$

Ans. **(ABCD)**

Sol. $(a, f(a)) \equiv r$

f'(x) be differentiation of f(x) equation of tangent

$(y - f(a)) = f'(a)(x - a)$

put x = 0

$y - f(a) = -af'(a)$

$y = f(a) - af'(a)$

$y_p = (0, f(a) - af'(a))$

$py_p = \sqrt{a^2 + (af'(a))^2} = 1$

$a^2 + a^2 (f'(a))^2 = 1$

$(f'(a))^2 = \frac{1-a^2}{a^2}$

$\int (f'(x)) = \pm\int\sqrt{\frac{1-x^2}{x^2}}$

put $\sqrt{1-x^2} = t$

$$\Rightarrow \qquad y = \pm\int\frac{-t^2dt}{1-t^2} = \pm\left(t - \frac{1}{2}\ell n\left|\frac{1+t}{1-t}\right|\right) + c = \pm\left(t - \frac{1}{2}\ell n\frac{(1+t)^2}{1-t^2}\right) + c = \pm\left(\sqrt{1-x^2} - \ell n\frac{1+\sqrt{1-x^2}}{x}\right) + c$$

$\Rightarrow \qquad$ Put x = 1 and y = 0 $\Rightarrow$ c = 0

12. Let $f : R \to R$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ (2/3)x^3 - 4x^2 + 7x - (8/3) & 1 \le x < 3 \\ (x-2)\ell n(x-2) - x + (10/3) & x \ge 3 \end{cases}$$

Then which of the following options is/are Correct ?

(A) f is onto

(B) f ' is not differentiable at x = 1

(C) f ' has a local maximum at x = 1

(D) f is increasing on $(-\infty, 0)$

Ans. **(ABC)**

Sol. $$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1 & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{9} & 1 \le x < 3 \\ (x-2)\ln(x-2) - x + \frac{10}{3} & x \ge 3 \end{cases}$$

$$f'(x) = \begin{cases} 5(x+1)^4 - 2 & x < 0 \\ 2x - 1 & 0 \le x < 1 \\ 2x^2 - 8x + 7 & 1 \le x < 3 \\ \ln(x-2) & x \ge 3 \end{cases}$$

$x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1$ takes value between $-\infty$ to 1

Also $(x-2)\ln(x-2) - x + \frac{10}{3}$ takes value between $\frac{1}{3}$ to ∞

So, range of f(x) is R. So option (A) is correct

$f''(1^-) = 2$ and $f''(1^+) = -4$

so f'(x) is non-diff at x = 1 so option (B) is correct

f'(x) has local maxima at x = 1 so option (C) is correct

13. $I = \dfrac{2}{\pi}\displaystyle\int_{-\pi/4}^{\pi/4} \dfrac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then find $27I^2$.

$I = \dfrac{2}{\pi}\displaystyle\int_{-\pi/4}^{\pi/4} \dfrac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ तब $27I^2$ का मान ज्ञात कीजिए।

Ans. (4)

Sol. $I = \dfrac{2}{\pi}\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \dfrac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ ....(1)

by a + b – x property

$I = \dfrac{2}{\pi}\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \dfrac{dx}{(1+e^{-\sin x})(2-\cos 2x)} = \dfrac{2}{\pi}\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \dfrac{e^{\sin x}dx}{(1+e^{\sin x})(2-\cos 2x)}dx$ .....(2)

adding (1) and (2)

$2I = \dfrac{2}{\pi}\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \dfrac{(1+e^{\sin x})}{(1+e^{\sin x})(2-\cos 2x)}dx \Rightarrow \quad I = \dfrac{1}{\pi}\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \dfrac{1}{2-(2\cos^2 x-1)}dx = \dfrac{1}{\pi}\displaystyle\int_{-\frac{\pi}{4}}^{\pi/4} \dfrac{\sec^2 x}{3\sec^2 x-2}dx$

put $\tan x = t, \sec^2 x dx = dt$

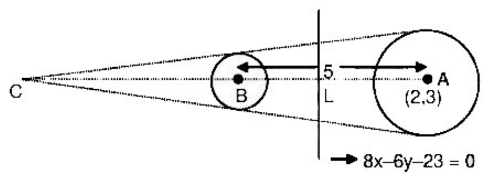
$= \dfrac{2}{\pi}\displaystyle\int_0^1 \dfrac{dt}{3t^2+1} = \dfrac{2}{3\pi}\dfrac{1}{\left(\frac{1}{\sqrt{3}}\right)}\left(\tan^{-1}\left(\dfrac{t}{1/\sqrt{3}}\right)\right)_0^1 = \dfrac{2}{\sqrt{3}\pi}\left(\tan^{-1}(\sqrt{3})-\tan^{-1}(0)\right) = \dfrac{2}{\sqrt{3}\pi}\left(\dfrac{\pi}{3}\right) = \dfrac{2}{3\sqrt{3}}$

Now $27I^2 = 27 \times \dfrac{4}{27} - 4$

14. Let the point B be the reflection of the point A(2, 3) with respect to the line $8x - 6y - 23 = 0$. Let T_A and T_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles T_A and T_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is

Ans. **(10)**

Sol.



$$AL = \left|\frac{16-18-23}{10}\right| = \frac{5}{2}$$

$$\frac{CB}{CA} = \frac{1}{2}$$

$$\frac{CA-5}{CA} = \frac{1}{2}$$

$CA = 10$

15. If (a, d) denotes an A.P. with first term a and common difference d. If the A.P. formed by intersection of three A.P's given by (1, 3), (2, 5) and (3, 7) is a new A.P. (A,D). Then the value of A + D is

Ans. **(157)**

Sol. First series is {1, 4, 7, 10, 13,........}

Second series is {2, 7, 12, 17, ...}

Third series is {3, 10, 17, 24, ...}

See the least number in the third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5.

$\Rightarrow$ 52 is the least number of third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5

Now, A = 52

D is L.C. M. of (3, 5, 7) = 105

$\Rightarrow A + D = 52 + 105 = 157$

16. Let S be the set of matrices of order 3×3, such that all elements of the matrix belong to $\{0, 1\}$. Let $E_1 = \{A \in S : |A| = 0\}$; where $|A|$ denotes determinant of matrix A.
$E_2 = \{A \in S : \text{Sum of elements of } A = 7\}$. Find $P(E_1/E_2)$

Ans. **(0.50)**

Sol. E_2 : Sum of elements of A = 7 $\Rightarrow$ These are 7 ones and 2 zeros

Number of such matrices = ${}^9C_2 = 36$.

Out of all such matrices ; E_1 will be those when both zeros lie in the same row or in the same colume

eg. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$n(E_1 \cap E_2) = 2 \times {}^3C_2 \times {}^3C_2 = 18$

$\uparrow \quad \uparrow$

So $n(E1/E2) = \dfrac{n(E_1 \cap E_2)}{n(E_2)} = \dfrac{18}{36} = \dfrac{1}{2}$

17. Equation of three lines $\vec{r} = \lambda \hat{i}$; $\vec{r} = \mu(\hat{i} + \hat{j})$; $\vec{r} = \gamma(\hat{i} + \hat{j} + \hat{k})$ and a plane $x + y + z = 1$ are given then area of triangle formed by point of intersection of line and plane is Δ , then $(6\Delta)^2$ equals

Ans. **(0.75)**

Sol. Put $(\lambda, 0, 0)$ in $x + y + z = 1$ $\Rightarrow \lambda = 1 \Rightarrow P(1, 0, 0)$

Put $(\mu, \mu, 0)$ $\Rightarrow 2\mu = 1$ $\Rightarrow Q\left(\dfrac{1}{2}, \dfrac{1}{2}, 0\right)$

Put (γ, γ, γ) $\Rightarrow \gamma = \dfrac{1}{3}$ $\Rightarrow R\left(\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}\right)$

Area of triangle PQR = $\dfrac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \dfrac{1}{2}\left|\left(\dfrac{\hat{i} - \hat{j}}{2}\right) \times \left(\dfrac{2\hat{i} - \hat{j} - \hat{k}}{3}\right)\right| = \dfrac{1}{12}\left|\hat{i} + \hat{j} + \hat{k}\right| = \dfrac{\sqrt{3}}{12} \Rightarrow (6\Delta)^2 = 0.75$

18. That $\omega \neq 1$ be a cube root of unity. Then minimum value of set $\{|a + b\omega + c\omega^2|^2$; a, b, c are distinct non zero integers$\}$ equals ______.

Ans. **(3)**

Sol.. $|a + b\omega + c\omega^2|^2 = a^2 + b^2 + c^2 - ab - bc - ca = \dfrac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

it will be minimum when a = 1, b = 2, c = 3

so minimum value is 3.